Partial Differential Equation Toolbox 1

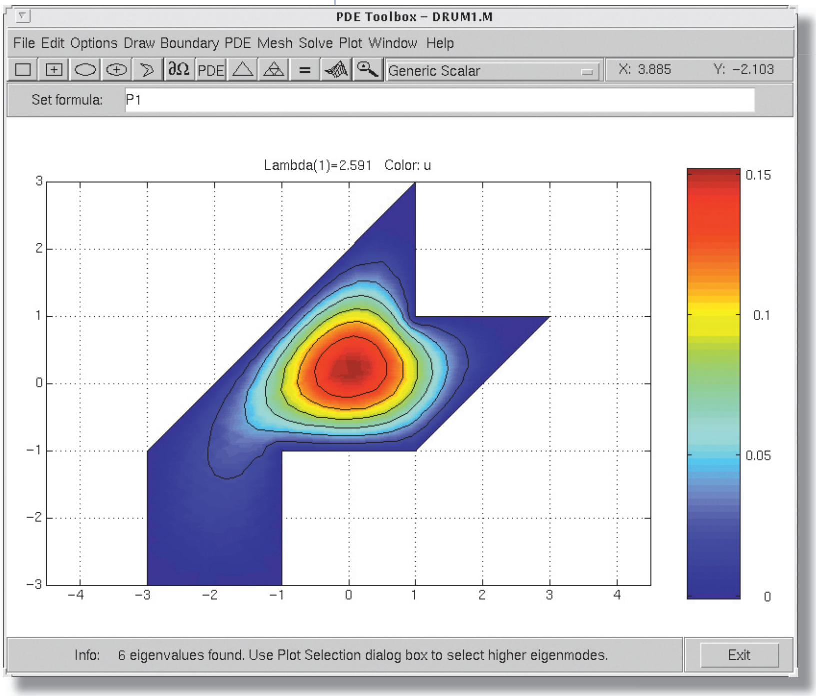
Solve and analyze partial differential equations

The Partial Differential Equation Toolbox extends the MATLAB® technical computing environment with tools for the study and solution of partial differential equations (PDEs) in two-space dimensions (2-D) and time. A set of command-line functions and a graphical user interface let you preprocess, solve, and postprocess generic 2-D PDEs using the Finite Element Method (FEM).

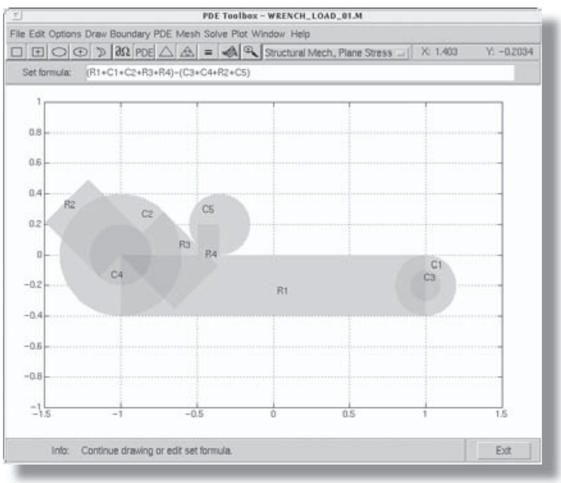
The toolbox also provides modeling capabilities for a broad range of applications, including heat transfer, electromagnetics, structural mechanics, and diffusion.

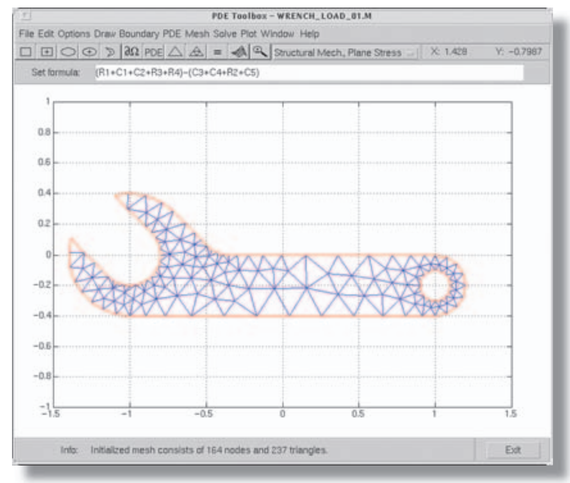
KEY FEATURES

- Graphical interface for pre- and postprocessing 2-D PDEs
- Automatic and adaptive meshing
- Geometry creation using a constructive solid geometry paradigm
- Boundary condition specification: Dirichlet, generalized Neumann, and mixed
- Flexible coefficient and PDE problem specification using MATLAB syntax
- Fully automated mesh generation and refinement
- Nonlinear and adaptive solvers to handle systems with multiple dependent variables
- Simultaneous visualization of multiple solution properties, FEM-mesh overlays, and animation



First eigenfunction of a drum membrane. The graphical user interface lets you visualize different solution properties using color, height, and vector fields.





Working With the Partial Differential Equation Toolbox

The Partial Differential Equation Toolbox lets you work in six modes from the graphical user interface or the command line. Each mode corresponds to a step in the process of solving PDEs using the Finite Element Method.

- **Draw** mode lets you create Ω, the geometry, using the constructive solid geometry (CSG) model paradigm. The graphical interface provides a set of solid building blocks (square, rectangle, circle, ellipse, and polygon) that can be combined to define complex geometries.
- **Boundary** mode lets you specify conditions on different boundaries or remove subdomain borders.
- **PDE** mode lets you select the type of PDE problem and the coefficients *c, a, f,* and *d*. By specifying the coefficients for each subdomain independently, you can represent different material properties.
- **Mesh** mode lets you control the fully automated mesh generation and refinement process.
- **Solve** mode lets you invoke and control the nonlinear and adaptive solver for elliptic problems. For parabolic and hyperbolic PDE problems, you can specify the initial values and obtain solutions at specific times. For the eigenvalue solver, you can define the interval over which to search for eigenvalues.
- **Plot** mode lets you select from different plot types, including surface, mesh, and contour. You can simultaneously visualize multiple solution properties using color, height, and vector fields. The FEM mesh can be overlaid on all plots and shown in the displaced position. For parabolic and hyperbolic equations, you can animate the solution as it changes with time.

Defining and Solving Partial Differential Equations

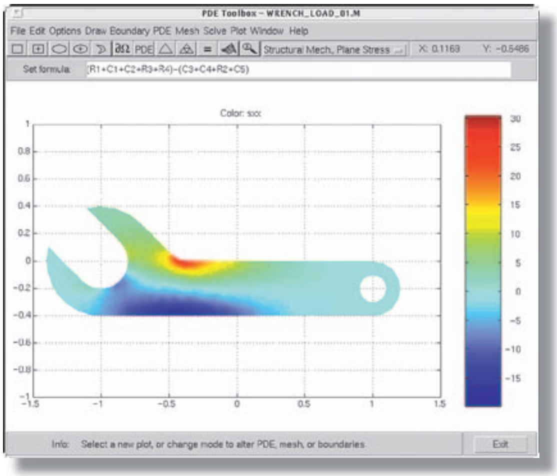
With the Partial Differential Equation Toolbox, you can define and numerically solve different types of PDEs, including elliptic, parabolic, hyperbolic, eigenvalue, nonlinear elliptic, and systems of PDEs with multiple variables.

Elliptic PDE

The basic scalar equation of the toolbox is the elliptic PDE

$$-\nabla \cdot (c\nabla u) + au = f \text{ in } \Omega$$

where ∇ is the vector $(\partial/\partial x, \partial/\partial y)$, and c is a 2-by-2 matrix function on Ω, the bounded planar domain of interest. c, a, and f can be complex valued functions of x and y.



Using the graphical user interface to define the complex geometry of a wrench, generate a mesh, and analyze it for a given load configuration.

Parabolic, Hyperbolic, and Eignevalue PDEs

The toolbox can also handle the parabolic PDE

$$d\frac{\partial u}{\partial t} - \nabla \cdot (c\nabla u) + au = f$$

the hyperbolic PDE

$$d\frac{\partial^2 u}{\partial t^2} - \nabla \cdot (c\nabla u) + au = f$$

and the eigenvalue PDE

$$-\nabla \cdot (c\nabla u) + au = \lambda du$$

where d is a complex valued function on Ω and λ is the eigenvalue. For parabolic and hyperbolic PDEs, c, a, f, and d can be complex valued functions of x, y, and t.

Nonlinear Elliptic PDE

A nonlinear Newton solver is available for the nonlinear elliptic PDE

$$-\nabla \cdot (c(u)\nabla u) + a(u)u = f(u)$$

where the coefficients defining c, a, and f can be functions of x, y, and the unknown solution u. All solvers can handle the PDE system with multiple dependent variables

$$-\nabla \cdot (c_{11}\nabla u) - \nabla \cdot (c_{12}\nabla v) + a_{11}u + a_{12}v = f_1$$

$$-\nabla \cdot (c_{21}\nabla u) - \nabla \cdot (c_{22}\nabla v) + a_{21}u + a_{22}v = f_2$$

You can handle systems of dimension two from the graphical user interface. An arbitrary number of dimensions can be handled from the command line. The toolbox also provides an adaptive mesh refinement algorithm for elliptic and nonlinear elliptic PDE problems.

Handling Boundary Conditions

The following boundary conditions can be handled for scalar u:

- Dirichlet:

$$hu = r$$

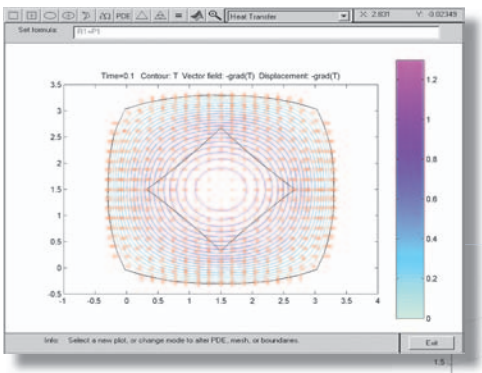
on the boundary $\partial\Omega$

- Generalized Neumann:

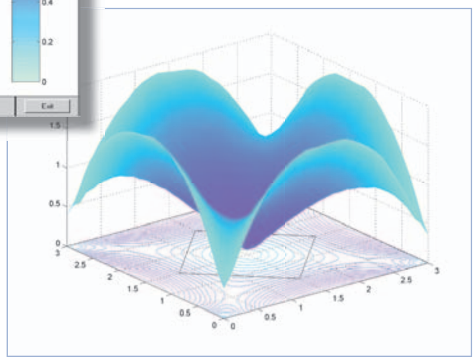
$$\vec{n} \cdot (c\nabla u) + qu = g$$

on $\partial\Omega$

where $\vec{n}$ is the outward unit normal and g, q, h, and r can be complex valued functions of x and y defined on $\partial\Omega$. For the nonlinear PDE, the coefficients may depend on u. For time-dependent problems, the coefficients may also depend on t. For PDE systems, Dirichlet, generalized Neumann, and mixed boundary conditions are supported.



Visualization tools provide multiple ways to plot results. A contour plot with gradient arrows shows the temperature and heat flux. The temperature gradient is displayed using 3-D plotting tools.



Toolbox Application Modes

The Partial Differential Equation Toolbox graphical interface includes a set of application modes for common engineering and science problems. When you select a mode, PDE coefficients are replaced with application-specific parameters, such as Young's modulus for problems in structural mechanics. Available modes include:

- Structural Mechanics - Plane Stress
- Structural Mechanics - Plane Strain
- Electrostatics
- Magnetostatics
- AC Power Electromagnetics
- Conductive Media DC
- Heat Transfer
- Diffusion

The boundary conditions are altered to reflect the physical meaning of the different boundary condition coefficients. The plotting tools lets you visualize the relevant physical variables for the selected application.

Required Products

MATLAB

Related Products

Optimization Toolbox. Solve standard and large-scale optimization problems.

Statistics Toolbox. Apply statistical algorithms and probability models.

Symbolic Math Toolbox *and* **Extended Symbolic Math Toolbox.** Perform computations using symbolic mathematics and variable-precision arithmetic.

Platform and System Requirements

For platform and system requirements, visit *www.mathworks.com/products/pde* ■

For demos, application examples, tutorials, user stories, and pricing:

- Visit **www.mathworks.com**
- Contact The MathWorks directly

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