

# Robust Control Toolbox 3

Design robust controllers for plants with uncertain parameters and unmodeled dynamics

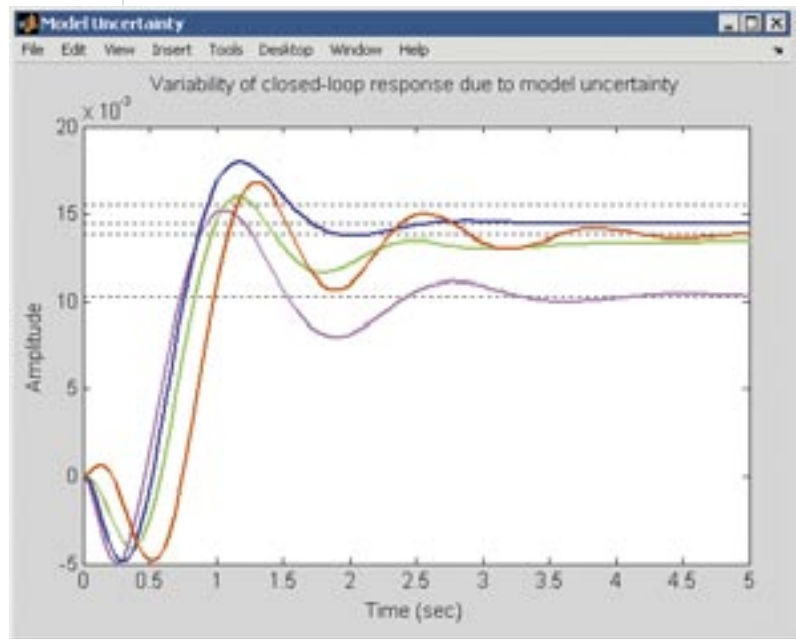
The Robust Control Toolbox provides tools for systematically factoring model uncertainty into your design to ensure consistent controller performance on the real plant. These tools let you quickly identify worst-case scenarios and automatically generate controllers with reduced sensitivity to parameter variations and modeling errors.

The toolbox extends the Control System Toolbox with tools that bridge classical control design methods and advanced robust control techniques. It includes algorithms to quantify uncertainty in your model, analyze its impact on control system performance, design robust controllers with guaranteed performance on the real plant, and reduce the complexity of plant and controller models. These algorithms are applicable to single-loop (SISO) and multi-loop (MIMO) control systems.

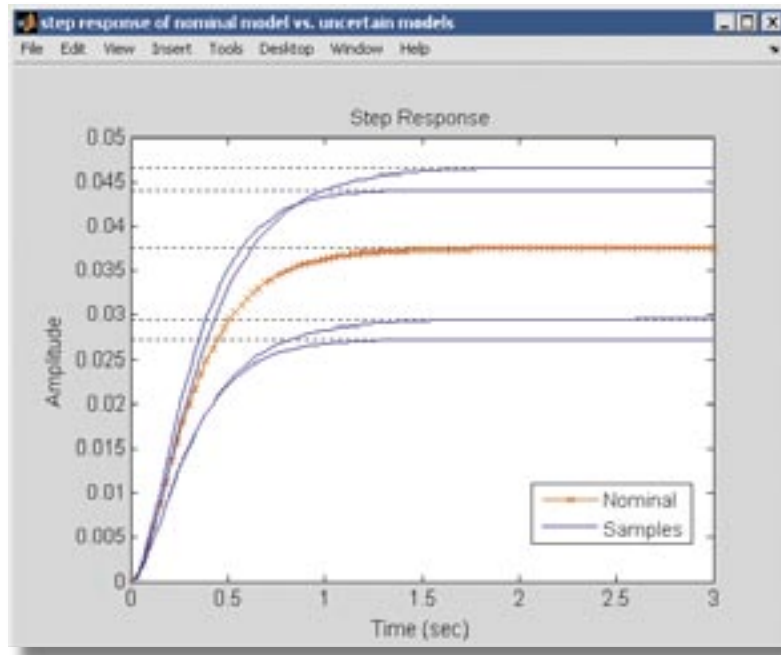


## KEY FEATURES

- $\mu$ -analysis and LMI-based techniques for analyzing the robustness of MIMO control systems
- Algorithms for frequency-domain loop shaping of MIMO open-loop responses
- $H_\infty$  and  $\mu$ -synthesis techniques for robust control system design
- Approximation algorithms for model order reduction
- General-purpose LMI solvers (feasibility, minimization of linear objectives, and generalized eigenvalue minimization)



Closed-loop responses of uncertain models.



A comparison of the step response in a nominal model and uncertain models.

### Working with the Robust Control Toolbox

A plant model is rarely a fully accurate description of the real plant. Neglected dynamics, approximately known parameters, and changes in operating conditions all contribute to plant-modeling errors that can jeopardize controller performance. A controller is “robust” when it maintains its performance regardless of plant modeling errors.

The Robust Control Toolbox provides a systematic approach to designing robust, multivariable feedback control systems. This approach involves the following steps:

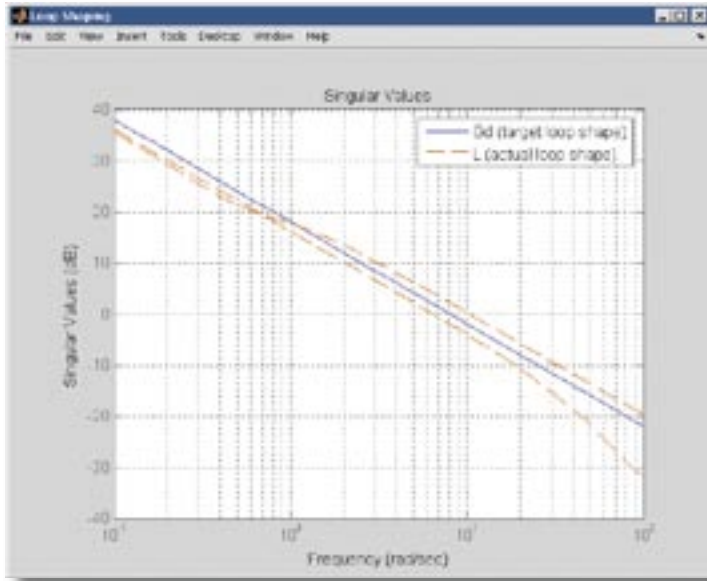
- Modeling and quantifying plant uncertainty
- Performing robustness analysis
- Synthesizing robust, multivariable controllers
- Reducing controller and plant model order

### Modeling and Quantifying Plant Uncertainty

With the Robust Control Toolbox, you can capture not only the typical, or “nominal” behavior of your plant, but also the amount of uncertainty and variability. Plant model uncertainty can result from:

- Model parameters with approximately known or varying values
- Neglected or poorly known dynamics, such as high-frequency dynamics
- Changes in operating conditions
- Linear approximations of nonlinear behaviors
- Estimation errors in a model identified from measured data

The toolbox lets you build detailed uncertain models by combining nominal dynamics with uncertain elements, such as an uncertain parameter or neglected high-frequency dynamics. By quantifying the level of uncertainty in each element, you can capture the overall fidelity and variability of your plant model. You can then analyze how each uncertain element affects performance and identify worst-case combinations of uncertain element values.



Loop shaping tools automate frequency-domain design of the open-loop response.

### Loop Shaping

Performance and robustness requirements can often be expressed in terms of the open-loop response gain. For example, high gain at low frequencies reduces steady-state offsets and improves disturbance rejection. Similarly, high-frequency roll-off improves stability where the plant model is poor. Loop shaping aims at designing a controller that achieves a particular loop shape.

The Robust Control Toolbox provides tools to automate computing controllers that best match user-defined loop shape. In MIMO systems, these tools operate on the singular values of the open-loop response to increase or decrease the gain in all input/output directions.

### $H_{\infty}$ and $\mu$ -Synthesis

The Robust Control Toolbox provides several algorithms for synthesizing robust MIMO controllers directly from closed-loop specifications. These specifications may include point-to-point gain limitations, frequency-dependent gain constraints, sensitivity constraints, pole placement objectives, and LQG-like cost functions. Powerful  $\mu$ -synthesis algorithms can optimize controller performance in the presence of model uncertainty, ensuring effective performance under all realistic scenarios. Together, these tools provide unique insight into the performance limits of your control architecture, as well as quick first-cut compensator designs.

### Performing Robustness Analysis

Using the Robust Control Toolbox, you can analyze the effect of plant model uncertainty on the closed-loop stability and performance of the control system. In particular, you can determine whether your control system will perform adequately over your entire operating range, and what source of uncertainty is most likely to jeopardize performance.

You can randomize the model uncertainty to perform Monte-Carlo analysis. Alternatively, you can use more direct tools based on  $\mu$ -analysis and LMI optimization. These tools use sophisticated algorithms to identify worst-case scenarios without exhaustive simulation.

The Robust Control Toolbox provides tools to assess:

- Worst-case gain/phase margins one loop at a time
- Worst-case stability margins taking loop interactions into account

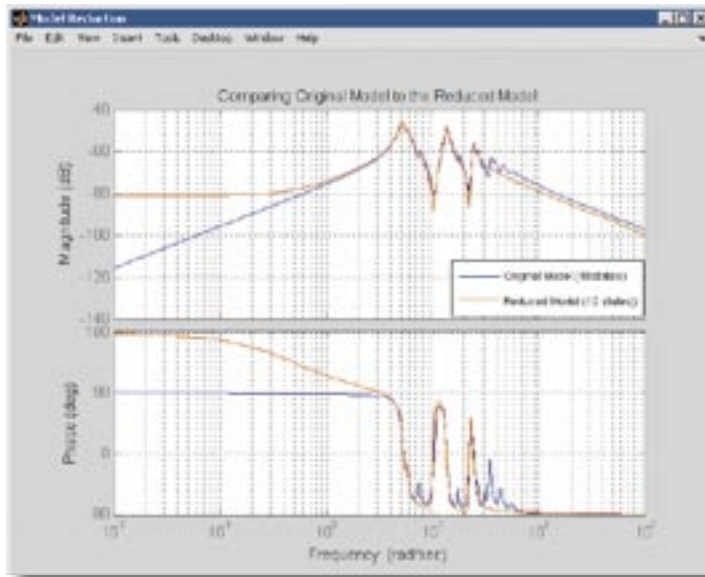
- Worst-case gain between any two points in the closed-loop system
- Worst-case sensitivity to external disturbances

These tools also provide sensitivity information to help you identify which uncertain elements contribute most to performance degradation. You can then determine whether a more accurate model, tighter manufacturing tolerances, or a better sensor would most improve control system robustness.

### Synthesizing Robust Multivariable Controllers

The Robust Control Toolbox provides a variety of controller synthesis algorithms based on loop shaping,  $H_{\infty}$  or  $\mu$ -synthesis, and LMI techniques. These algorithms are applicable to SISO and MIMO control systems. MIMO controller synthesis does not involve sequential loop closure, and is therefore well suited to multi-loop control systems with strong loop interaction and challenging I/O decoupling requirements.

A comparison of the original and reduced-order models.



### Reducing Controller and Plant Model Order

The Robust Control Toolbox provides tools for reducing the order (number of states) of a plant or controller model while preserving its essential dynamics. Detailed first-principles or finite-element plant models may have a high order. Similarly,  $H_\infty$  or  $\mu$ -synthesis algorithms tend to produce high-order controllers with superfluous states. In both cases, model reduction lets you develop approximate plant and controller models that are reliable and cost-effective to implement.

The toolbox offers state-of-the-art algorithms for extracting reduced-order models while controlling the approximation error. The reduction techniques and error-bound calculations are based on Hankel singular values of the system, which measure the energy of the states. By retaining high-energy states and ignoring low-energy states, the reduced model preserves the essential features of the original model. You can use the absolute or relative approximation error to select the order, and use frequency-dependent weights to determine the level of accuracy appropriate to the frequency ranges.

### Required Products

MATLAB  
Control System Toolbox

### Related Products

Simulink. Simulation and Model-Based Design

System Identification Toolbox. Create linear dynamic models from measured input - output data

For more information on related products, visit [www.mathworks.com/products/robust](http://www.mathworks.com/products/robust)  
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For platform and system requirements, visit [www.mathworks.com/products/robust](http://www.mathworks.com/products/robust) ■

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