Quadratic Programmig Solution to Emission and Economic Dispatch Problems

R M S Danaraj, *Non-member*

Dr F Gajendran, *Non-member*

This paper presents a new and efficient way of implementing quadratic programming to solve the economic and emission dispatch problems. Economic load dispatch (ELD), minimum emission dispatch (MED), combined economic emission dispatch (CEED) and emission controlled economic dispatch (ECED) are solved using the proposed method. Transformation of variables technique along with quadratic programming is applied recursively to solve both problems. The advantage of this method is its robustness to find the global minimum for all the problems. The algorithm is tested on a test system and compared with genetic algorithm and hybrid genetic algorithm. The results clearly demonstrate the effectiveness of the proposed method.

Keywords : Economic load dispatch (ELD); Minimum emission dispatch (MED); Combined economic and emission dispatch (CEED); Emission constrained economic dispatch (ECED); Transformation of variables technique; Quadratic programming

NOTATION

a_i, b_i, c_i	:	fuel cost coefficients of 'i'th plant
B_{mn}	:	loss coefficient metrics
d_i, e_i, f_i	:	emission coefficients
L_i	:	lower power limit of 'i'th power plant
N	:	no of plants
P_d	:	real power demand on the system
P_i	:	real power generation of 'i'th power plant
U_i	:	upper power limit of 'i'th power plant

INTRODUCTION

The operation and planning of a power system is characterized by maintaining a high degree of economy and reliability. The plants have to meet the demand and the transmission losses for minimum cost while meeting the constraints (economic load dispatch). Traditionally electric power plants are operated on the basis of least fuel cost strategies and very little attention is paid on the pollution produced by these plants.

Recently, passage of the 'Clean Air Act Amendment of 1990' and its acceptance by all the nations has forced the utilities to modify their operating strategies to meet the rigorous environment standards set by this legislation. Thus the modern operational strategies of the generating plants now include reduction of pollution level up to a safe limit set by environmental regulating authority, in addition to minimum fuel cost strategies and transmission security objective.

Major part of the power generation is due to fossil fired plants and their emission contribution cannot be neglected. Fossil fired electric power plants use coal, oil, gas, or combination thereof as primary energy resource and produce atmospheric emission whose nature and quantity depend upon fuel type and its quality. Coal produce particulate matter such as ash and gaseous pollutants such as CO_2, NO_X (oxides of nitrogen) etc. Therefore there is a need to reduce the emission from these fossil fired plants either by design or by operational strategies.

The characteristics of emissions of various pollutants are different and are usually highly nonlinear. This increases the complexity and non-monotonocity of the combined emission and economic dispatch (CEED) problem. Many authors have addressed the economic dispatch problem. EL-Keib and Hart[1] have presented a general formulation of the environmental constrained economic dispatch (ECED) problem, which is linear programming and uses gradient projection method to guarantee feasibility of the solution. K Srikrishna and C Palanichamy[2] have proposed a method for combined emission and economic dispatch using price penalty factor. R Ramaratnam[3] developed a technique to add emission constraints to the standard classical economic dispatch problem. S Baskar *et al*[4] have applied hybrid genetic algorithm to solve the problem of CEED and ECED, Dr S L Surana and P S Bhati[5] also tried with GA to solve ECED with better results. It is well known that GA consumes more time and not certain to find the global minimum all the time.

In this paper Quadratic program along with Transformation

R M S Danraj and Dr F Gajendran are with Research and Development, Sri Krishna College of Engineering and Technology, Coimbatore 641 008.

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of variables technique is used to solve ELD, MED CEED and ECED problems. Quadratic programming is an effective tool to find global minimum for optimisation problems having quadratic objective and linear constraints. The objective function is quadratic for both cases but the constraints are not linear. The constraints are linearised by transformation of variable technique and the quadratic programming is applied recursively till the convergence is reached. It is compared with genetic algorithm[5], real coded genetic algorithm[4] and hybrid genetic algorithm[4]. The results clearly demonstrate the effectiveness and robustness of this method over Hybrid GA and GA.

PROBLEM FORMULATION

There are so many ways for including emission into the formulation of economic dispatch. One approach is combined economic and emission dispatch (CEED), which is formulated as a multi-objective optimisation problem, which should minimize both, fuel cost and emission subject to meet the demand and losses. Another approach is emission controlled economic dispatch (ECED), which is minimizing the economy subject to that particular emission limit for particular demand.

Combined Emission and Economy Dispatch

The combined economic and emission dispatch problem can be formulated as[6]

$$\text{Minimize } f(FC, EC), \ni, \sum_{i=1}^{N} P_i = P_d + P_l, L_i \le P_i \le U_i \quad (1a)$$

$$FC = \sum_{i=1}^{N} a_i P_i^2 + b_i P_i + c_i \quad (1b)$$

$$EC = \sum_{i=1}^{N} d_i P_i^2 + e_i P_i + f_i \quad (1c)$$

$$p_1 = \sum_{i=1}^{N} \sum_{j=1}^{N} B_{ij} P_j P_i \quad (1d)$$

FC is the total fuel cost and EC is the total emission. The transmission losses P_1 can be found either from load flow or using B_{mn} coefficients. Though this method can incorporate both cases B_{mn} coefficients are used to calculate transmission losses in this paper. The multi objective optimisation problem is converted as single objective optimisation problem by using price penalty factor as follows

$$\text{Minimize } f(FC,\ EC) = \text{Minimize}(FC + h\,EC) \quad (2)$$

The price penalty factor or each plant can be found for a particular demand as follows

1. The ratio between the average fuel cost and the average emission of maximum power capacity of that plant is found

$$h_i = FC_i(U_i)/EC_i(U_i),\ i = 1, 2, n \quad (3)$$

2. Based on the value of price penalty factor found the plants are arranged in ascending order
3. The maximum capacity of each unit (U_i) is added one at a time, starting from the smallest h_i, unit until $\sum P_i \ge P_d$
4. At this stage h_i, associated with the last unit in the process is the price penalty factor 'h', Rs/Kg for the given load demand.

Emission Controlled Economic Dispatch (ECED)

The main objective of the ECED problem is to determine the most economical allocation of plants in such away to meet the demand and losses while keeping the emission level at allowable limit For ECED, FC is to be minimized subject to the power balance constraint equation (1a) and emission limit constraint. It can be expressed as equation (4).

$$\text{Minimize } f(FC), \ni, \sum_{i=1}^{N} P_i = P_d + P_l,$$
$$L_i \le P_i \le U_i, EC \le E_{\text{limit}} \quad (4)$$

Where E_{limit} is the total emission limit over the system.

QUADRATIC PROGRAMMING

Quadratic Programming is an effective optimisation method to fid the global solution if the objective function is quadratic and the constraints are linear. It can be applied to optimisation problems having non-quadratic objective and non-linear constraints by approximating the objective to quadratic function and the constraints as linear. For all the four problems the objective is quadratic but the constraints are also quadratic so the constraints are to be made linear. Transformation of variables technique[7] is incorporated for making the constraints linear. This is explained as follows.

1. Put $P_i = L_i + (U_i - L_i),\ X_i$, where $0 < X_i < 1$ in the objective function and the constraints.
2. Make the constraints linear by neglecting the second order terms for the constraints
3. Apply QP to solve the optimisation problem find the solution vector [P].

Table 1 Optimal allocation of economic load dispatch by proposed method

Pd, MW	P1, MW	P2, MW	P3, MW	P4, MW	P5, MW	P6, MW
700	27.861	10.000	116.826	119.588	231.474	213.729
1100	47.705	37.681	220.240	201.126	325.000	315.000

Table 2 Comparison of results for economic load dispatch

Demand	Performance	GA[4]	Hybrid GA[4]	Proposed[12] Method
700	FC, Rs/hr	36912.240	37137.960	36899.570
	EC, Rs/hr	501.013	489.550	502.030
	PI, MW	19.430	23.124	19.478
1100	FC, Rs/hr	57870.530	-	57834.560
	EC, Rs/hr	1231.843	-	1232.660
	PI, MW	46.850	-	46.890

Table 3 Optimal allocation of minimum emission dispatch by proposed method

Pd, MW	P1, MW	P2, MW	P3, MW	P4, MW	P5, MW	P6, MW
700	80.214	82.474	113.934	113.444	163.411	163.060
1100	125	150	178.602	177.126	255.914	254.824

Table 4 Comparison of results for minimum emission dispatch

Demand	Performance	GA[5]	Hybrid GA[4]	Proposed Method
700	FC, Rs/hr	38100.990	38186.400	38091.948
	EC, Rs/hr	434.130	435.075	433.972
	Pl, MW	16.540	17.366	16.538
1100	FC, Rs/hr	60628.940	—	60600.630
	EC, Rs/hr	1022.195	—	1021.930
	Pl, MW	41.470	—	41.467

Table 5 Optimal power dispatch using QP for CEED Problem

Pd, MW	P1, MW	P2, MW	P3, MW	P4, MW	P5, MW	P6, MW
500	33.907	26.850	89.793	90.356	135.590	132.820
700	62.278	61.739	119.993	119.993	178.951	175.471
900	93.000	98.400	150.120	148.850	220.310	218.400

4. Now set the lower limit equal to the solution vector that is $L = [P]$.
5. Repeat the steps 1, 2, 3, and 4 till the convergence is reached.

SIMULATION RESULTS

A test system having six thermal units is considered for simulation. The plant data is given in Appendix I. The proposed method is applied for CEED for demands 500 MW. 700MW and 900MW and it is compared with real coded GA[4] and Hybrid GA[4]. The optimal allocations and comparisons with other methods for ELD and minimum emission are given in Tables 1-4. The optimal allocation for CEED and the comparison are given in Tables 5 and 6. This method is applied for ECED for demands of 700MW and 1100MW and compared with GA[5] and Hybrid GA[4]. The results are given in Tables 7 and 8. From the results it is proved that QP outperforms GA and Hybrid GA in all aspects.

Table 6 Comparison of results for combined economic emission dispatch

Demand	h, Rs/kg	Performance	GRA[4]	Hybrid GA[4]	Proposed[12]
500	43.898	FC, Rs/hr	27638.300	27695.000	27606.470
		EC, Rs/hr	263.472	263.370	262.400
		Pl, MW	10.172	10.135	8.932
		Total	39258.080	39257.500	39149.380
700	44.788	FC, Rs/hr	37640.370	37640.400	37488.580
		EC, Rs/hr	439.979	439.978	439.720
		Pl, MW	18.521	18.517	17.054
		Total	57346.190	57346.100	57171.450
		Cost Rs/hr			
900	47.822	FC, Rs/hr	48567.750	48567.500	48330.310
		EC, Rs/hr	694.169	694.172	693.600
		Pl, MW	29.725	29.718	28.007
		Total	81764.450	81764.400	81499.420
		Cost Rs/hr			

Table 7 Optimal power dispatch using QP for ECED problem

Pd, MW	P1, MW	P2, MW	P3, MW	P4, MW	P5, MW	P6, MW
700	56.437	53.969	121.659	121.573	183.610	180.046
1100	101.497	112.386	189.256	185.517	278.602	275.580

Table 8 Comparison of results-emission constrained economic dispatch

Demand	Performance	Emission Limit	Genetic[5] Algorithm	Hybrid[4] Genetic Algorithm	Proposed[12] Method
700	FC, Rs/hr	—	38389.410	—	37329.700
	EC, Rs/hr	444	442.551	—	444.000
	Pl, MW	—	17.220	—	17.293
1100	FC, Rs/hr	—	59207.934	59529.300	59141.150
	EC, Rs/hr	1060	1058.586	1060.000	1060.000
	Pl, MW	—	42.800	45.986	42.840

CONCLUSION

In this Paper, a new way of implementing the Quadratic programming to solve economic as well emission problems was proposed. In order to prove the effectiveness of the proposed method it is applied to six plant system and compared with GA and Hybrid GA.It is observed that it faster and finding the best possible solution.

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APPENDIX 1

PLANT DATA

Fuel Cost Equations

$F_1 = 0.15247\ P_1^2 + 38.53973\ P_1 + 756.79886$

$F_2 = 0.10587\ P_2^2 + 46.15916\ P_2 + 451.32513$

$F_3 = 0.02803\ P_3^2 + 40.3965\ P_3 + 1049.9977$

$F_4 = 0.03546\ P_4^2 + 38.30553\ P_4 + 1243.5311$

$F_5 = 0.02111\ P_5^2 + 36.32782\ P_5 + 1658.569$

$F_6 = 0.01799\ P_6^2 + 38.27041\ P_6 + 1356.6592$

Emission Equations

$E_1 = 0.00419\ P_1^2 + .32767\ P_1 + 13.85932$

$E_2 = 0.00419\ P_2^2 + .32767\ P_2 + 13.85932$

$E_3 = 0.00683\ P_3^2 - 0.54551\ P_3 + 40.2669$

$E_4 = 0.00683\ P_4^2 - 0.54551\ P_4 + 40.2669$

$E_5 = 0.00461\ P_5^2 - 0.51116\ P_5 + 42.89553$

$E_6 = 0.00461\ P_6^2 - 0.51116\ P_6 + 42.89553$

Generating Capacity Limits

Plant	1	2	3	4	5	6
L_i	10	10	35	35	130	125
U_i	125	150	225	210	325	315

B_{mn} Coefficients in the Order of 10^{-4}

1.40	0.17	0.15	0.19	0.26	0.22
0.17	0.60	0.13	0.16	0.15	0.20
0.15	0.13	0.65	0.17	0.24	0.19
0.19	0.16	0.17	0.71	0.30	0.25
0.26	0.15	0.24	0.30	0.69	0.32
0.22	0.20	0.19	0.25	0.32	0.85

APPENDIX 2

Solution to Economic Dispatch by Quadratic Programming Incorporating Transformation of Variables Technique[7]

The ELD can be described as follows

$$\text{Minimize} \sum_{i=1}^{N} a_i P_i^2 + b_i P_i + c_i \ni, \sum_{i=1}^{N} P_i = P_d + P_l, L_i \le P_i \le U_i \tag{A1}$$

$$P_1 = \sum_{i=1}^{N} \sum_{j=1}^{N} B_{ij} P_j P_i \tag{A2}$$

Put $P_i = L_i + (U_i - L_i)X_i$ and neglect the second order terms in the constraints. Now the problem becomes typical quadratic programming problem with quadratic objective and linear constraints.

$$\text{Minimize} \sum_{i=1}^{N} A_i X_i^2 + B_i P_i + C_i \ni, \sum_{i=1}^{N} K_i X_i = P_d - \sum_{i=1}^{N} L_i + \sum_{j=1}^{N} \sum_{i=1}^{N} L_i L_j, \quad 0 \le X_i \le 1 \tag{A3}$$

Where

$$A_i = a_i (U_i - L_i)^2 \quad B_i = (2a_i L_i + b_i)(U_i - L_i) \quad C_i = a_i L_i^2 + b_i L_i + c_i \tag{A4}$$

$$K_i = (U_i - L_i)(1 - 2\sum_{j=1}^{N} B_{ij} L_j) \tag{A5}$$

After this transformation the solution vector [X] can be found using QP once the solution is found now the lower limit is made equal to the solution vector and the procedure is repeated till the desired convergence. It is found that it is finding the global minimum all the time for ELD from any starting point since it is a convex programming problem.