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The DC Superposition (DS) Method of Analog Circuit Analysis

Users of MATLAB are assumed to be well-versed in matrix analysis. It is further assumed that electrical engineers using MATLAB are at least somewhat familiar with state space methods. Hence introductory material on matrices and state space methods is omitted. We only note a big advantage of state space analysis is that we can get the dc output, ac frequency response, transient response, and the circuit transfer function, all from one initial analysis. Another advantage is that the matrices are real, not complex.

The matrix equations used in state space analysis are

$$\frac{dx}{dt} = Ax + Bu, \qquad y = Dx + Eu + G\frac{dx}{dt}$$

where A, B, D, E, and G are arrays, x is a column vector of the state variables, u is a column vector of inputs, and y is the output. Array G is often zero, but can be used to simplify the output equation. (See UsingGarray1.pdf, and UsingGarray2.pdf for examples.) With G = 0 and taking the Laplace Transform of the first equation and substituting into the second gives

$$y = D\left(sI - A\right)^{-1} Bu + Eu$$

Using N as the order of the circuit (number of L's and/or C's), M the number of inputs, and K the number of outputs, the arrays have the following dimensions:

Array	Rows	Columns
A	N	N
B	N	M
D	K	N
E	K	M
G	K	N
I	N	N
x	N	1
u	M	1
y	K	M

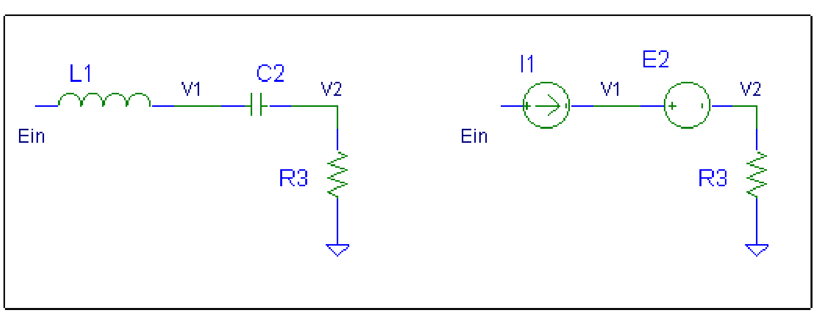
Then y is a transfer matrix with the dimensions {K M}. (I is an identity matrix.)

The first example to illustrate the method is a simple series LCR circuit.

The first step is to convert all inductors to ideal 1A current sources and all capacitors to ideal 1V voltage sources. This includes the input sources. The input can be other values, but for transfer function output is 1V.

Original Circuit

Converted Circuit



After this has been done, the analysis proceeds as follows:

1. Assign component and unity source values:

   L1 = 1mH   C2 = 0.253303uF   R3 = 5Ω   I1 = 1A   E2 = 1V
   Ein = 1, but can be other values.

2. Assign four circuit parameter constants N, M, U, & Y, that describe the circuit.

   N = number of capacitors and/or inductors in the circuit = 2 here.
   M = number of independent inputs. = 1
   U = number of unknown node voltages. = 2
   Y = the output node. = 2.
   K = the number of outputs, which can be up to U. Here we choose K = 1.

3. Write U+N DC equations for the circuit. No AC or Laplace transform equations are required.

   $$i_{C2} = I1, \quad V2 = I1 \cdot R3, \quad E2 = V1 - V2, \quad e_{L1} = Ein - V1$$

4. Rearrange the DC equations so that unknowns are on left-hand (LH) side, and knowns are on RH side:

   The first two equations are already in the correct form. For the last two:

   $$V1 - V2 = E2, \quad V1 + e_{L1} = Ein$$

5. The four unknowns are V1, V2, eL1, & iC2. As a memory aid, place these labels over a U+N square matrix A1, and fill in the coefficients from each equation (row).

The equations are repeated below followed by array A1 for the LH sides and column vector B1 for the RH sides.

$$i_{C2} = I1$$
$$\frac{V2}{R3} = I1$$
$$V1 - V2 = E2$$
$$V1 + e_{L1} = Ein$$

$$\begin{array}{cccc} V1 & V2 & eL1 & iC2 \end{array}$$

$$A1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & \frac{1}{R3} & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \qquad B1 = \begin{bmatrix} I1 \\ I1 \\ E2 \\ Ein \end{bmatrix}$$

6. Using superposition, expand B1 into N+M = 3 columns as array B2. The columns are labled I1, E2 and Ein. (Note that all rows and columns of A1 must have at least one non-zero element, and that all columns of B2 must have at least one non-zero element.)

$$
\begin{array}{cc}
 & \begin{array}{ccc} I1 & E2 & Ein \end{array} \\
B2 = & \begin{bmatrix} I1 & 0 & 0 \\ I1 & 0 & 0 \\ 0 & E2 & 0 \\ 0 & 0 & Ein \end{bmatrix}
\end{array}
$$

Create a N x N diagonal matrix P, with L1 and C2 in the same order as I1 and E2 in array B2.

$$P = \begin{bmatrix} L1 & 0 \\ 0 & C2 \end{bmatrix}$$

One of many advantages of this method is that IF DONE CORRECTLY UP TO THIS POINT, THE REMAINDER OF THE ANALYSIS DOES NOT CHANGE FROM CIRCUIT TO CIRCUIT; I.E., IT IS AUTOMATED, OR “CANNED”, AND HENCE GUARANTEED TO BE CORRECT.

The remainder of the canned procedure is given as MATLAB M-file statements. Remember, these do not change:

Solve for the superposed dc node voltages:

`V=A1\B2;` (In textbook math notation, $V = A1^{-1} \cdot B2$)

Extract the last N rows from V and label it H

```
H=V(U+1:U+N,1:N+M);
```

Solve for array AB which contains A in the first N columns and B in the last M columns:

```
AB=P\H;
```

Extract A and B from AB:

```
A=AB(1:N,1:N);B=AB(1:N,N+1:N+M)
```

Finally, extract D and E from V:

```
D=V(Y:Y,1:N);E=V(Y:Y,1:N+M)
```

7. DC Analysis

```
X=-A\B; % This gives an {N 1} vector X of the dc inductor
% current(s) and/or dc capacitor voltage(s)

Vdc = D*X+E; % DC output.
```

8. AC Analysis

Create identity matrix: I = eye(N). For a linear frequency sweep in Hz, assign beginning frequency BF, last frequency LF and frequency increment DF:

```
BF=5000      LF=15000     DF=100
```

See the M-file dsintro.m

The {U+N M} array V contains the superposed dc node voltages, the dc current through the 1V voltage sources (capacitors) and the dc voltage across the 1A current sources (inductors).

For this example circuit

$$V = \begin{bmatrix} 5 & 1 & 0 \\ 5 & 0 & 0 \\ -5 & -1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

The 1st row [5 1 0] is the node voltage V1 with I1, then E2, and lastly Ein activated separately With 1A of current thru 5Ω we have 5V. With only E2 = 1V, there is no current because I1 = 0 = open and V1 is merely the high side of E1. With Ein = 1V, again there is no current due to I1 = 0 and V1 is at ground potential.

This proceeds down the rows with the 2nd row being superposed voltages at V2, the 3rd row the superposed voltages across the 1A source (representing L1), and so forth. The last row is the superposed dc currents through E2 (representing C2).

To identify the V array coordinates, we can place I1 E2 Ein across the top and V1 V2 eL1 iC2 vertically on one side.

Array H is the last N = 2 rows of V, which are eL1 and iC2. The AB array (containing A and B) is obtained by multiplying H by the inverse of P.

Since Y = 2, D is the first two columns of the 2nd row (V2) which is [5 0]. E is the last column of this row or E = [0]. If Y were 1, the 1st row of V would be selected with D = [5 1] and E = [0].

Output of dsintro.m copied from the MATLAB Command Window:

```
V =
     5     1     0
     5     0     0
    -5    -1     1
     1     0     0

H =
    -5    -1     1
     1     0     0

AB =
        -5000        -1000         1000
  3.9478e+006            0            0
```

```
A =
        -5000         -1000
  3.9478e+006             0

B =
        1000
           0

D =
     5     0

E =
     0

L =
     -397.89 +     9992.1i
     -397.89 -     9992.1i

fo =
        10000

X =
     0
     1

Vdc =
     0
```