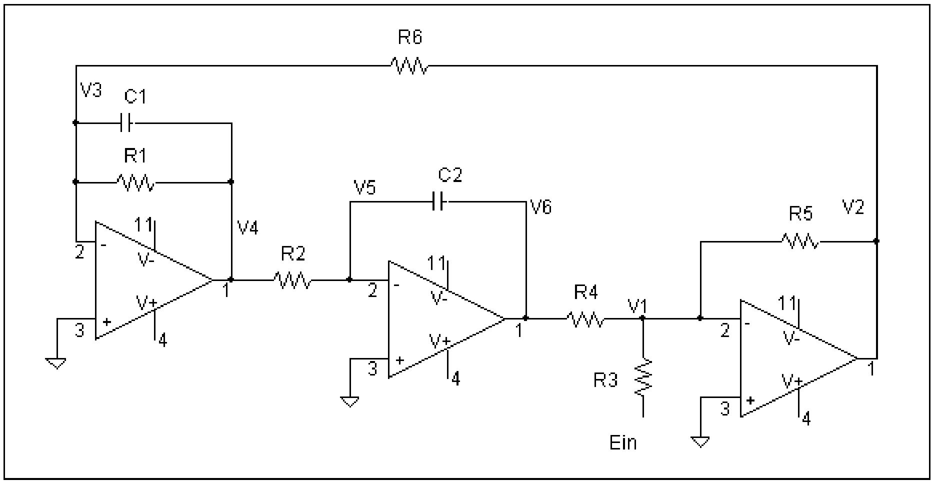
Tow-Thomas Biquad Bandpass Filter – DS Method (Three outputs using one analysis)



First we note that, due to the opamps, V1 = V3 = V5 = 0. Hence there are only three unknown nodes, the outputs at V2, V4, and V6. Hence dimension K = 3. (See introduction).

Here N = 2, U = K = 3, and M = 1. We will deal with the output nodes Y later.

At node V3 = 0:

$$\frac{V2 - V3}{R6} = i_{C1} + \frac{V3 - V4}{R1}, \quad \text{or} \quad \frac{V2}{R6} - i_{C1} + \frac{V4}{R1} = 0$$

From the schematic, output V2 is merely a 2-input inverting opamp, hence

$$V2 = -R5\left(\frac{V6}{R4} + \frac{Ein}{R3}\right), \quad \text{or} \quad V2 + \frac{R5 \cdot V6}{R4} = \frac{-R5 \cdot Ein}{R3}$$

At node V5 = 0:

$$\frac{V4}{R2} - i_{C2} = 0$$

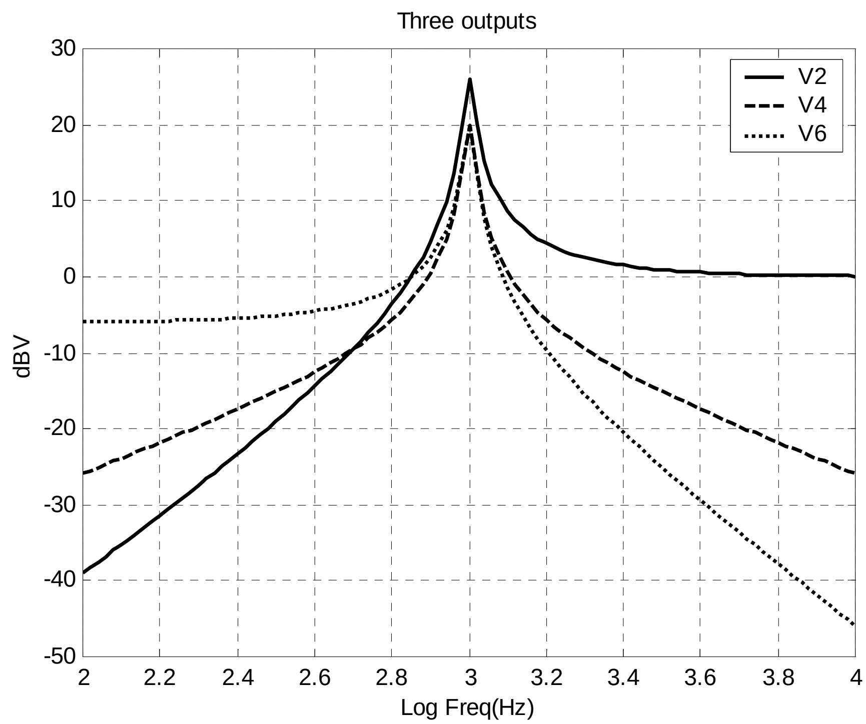
For the 1V ideal sources E1 and E2 replacing C1 and C2:

$$E1 = V3 - V4 = -V4, \quad E2 = V5 - V6 = -V6$$ (We now have the required U+N = 5 equations.)

A1 and B2 are:

$$
A1 = \begin{array}{c}
\begin{array}{ccccc} V2 & V4 & V6 & iC1 & iC2 \end{array} \\
\begin{bmatrix}
\frac{1}{R6} & \frac{1}{R1} & 0 & -1 & 0 \\
1 & 0 & \frac{R5}{R4} & 0 & 0 \\
0 & \frac{1}{R2} & 0 & 0 & -1 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix}
\end{array}, \qquad
B2 = \begin{array}{c}
\begin{array}{ccc} E1 & E2 & Ein \end{array} \\
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & \frac{-R5}{R3} \\
0 & 0 & 0 \\
-1 & 0 & 0 \\
0 & -1 & 0
\end{bmatrix}
\end{array}
$$

Plot from TowThomasds.m is shown below:



D and E are repeated here for use in showing Leverriers algorithm below. Note that they are dimension D = {K N} and E = {K M}, where K = 3, N = 2, and M = 1.

$$
D = \begin{bmatrix} 0 & 2 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad E = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}
$$

It was mentioned in the introduction that the circuit or system transfer function can be obtained using Leverrier's algorithm. (For a reference see D.M. Wiberg, *State Space and Linear Systems*, Schaum's Outline Series, 1971, p.102)

The equations used in Leverrier's algorithm are in the M-file TowThomasLev.m, and are repeated here for clarity.

$$F1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{N = 2 identity matrix}$$

$$T1 = \frac{-\mathrm{tr}\left(A \cdot F1\right)}{1}, \quad T1 = 314 \qquad \text{tr is the trace of the matrix, or the sum of diagonals.}$$

$$F0 = A \cdot F1 + T1 \cdot F1, \quad F0 = \begin{bmatrix} 0 & 6289 \\ -6289 & 314 \end{bmatrix}$$

$$T0 = \frac{-\mathrm{tr}\left(A \cdot F0\right)}{2}, \quad T0 = 3.956 \cdot 10^{7}$$

$$Y1 = D \cdot F1 \cdot B + E \cdot T1, \quad Y1 = \begin{bmatrix} -314 \\ 3145 \\ 0 \end{bmatrix}$$

$$Y0 = D \cdot F0 \cdot B + E \cdot T0, \quad Y0 = \begin{bmatrix} 0 \\ 0 \\ -1.978 \cdot 10^{7} \end{bmatrix}$$

Using a matrix polynomial for the numerator, the {K M} = {3 1} transfer matrix is

$$Vo(s) = \frac{E \cdot s^{2} + Y1 \cdot s + Y0}{s^{2} + T1 \cdot s + T0}$$

The numerator of Vo(s) is a quadratic as is the denominator. Hence it is a bi-quadratic transfer function or "biquad".

The plot from M-file TowThomasLev.m is shown below, identcal to the previous plot.

Text File Output (qbout.txt) from TowThomasLev.m

```
A =
      -314.47       6289.3
      -6289.3            0

B =
      -3144.7
            0

D =
     0     2
    -1     0
     0    -1

E =
    -1
     0
     0

T1 =
       314.47

F0 =
            0       6289.3
      -6289.3       314.47

T0 =
  3.9555e+007

Y1 =
      -314.47
       3144.7
            0


Y0 =
            0
            0
 -1.9778e+007
```

Identical Plot from TowThomasLev.m using Transfer Matrix