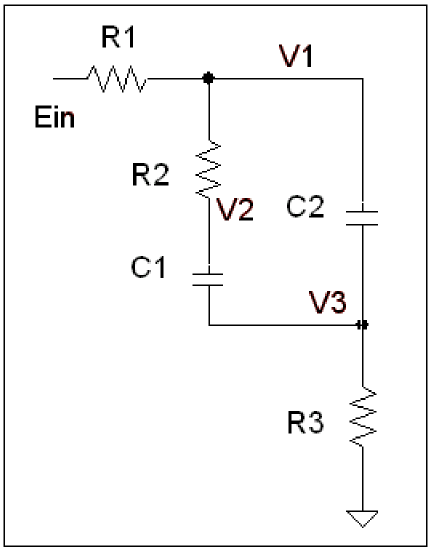
2/10/2007

Reduced Algebra (RA) Method With the G Array [1]

Passive Magnet Driver Circuit.



Notes: Subscripts are not used below to improve legibility.
This method is original with the author. See [1].

The goal is to put the circuit equations in the form f(eL,iC) = g(Ein,iL,vC) (1)

From the schematic: $\frac{\mathrm{Ein}-\mathrm{V1}}{\mathrm{R1}}=\mathrm{iC1}+\mathrm{iC2}$ (2)

Eliminating V1: $\mathrm{V1}=\mathrm{vC2}+\mathrm{V3}=\mathrm{vC2}+\left(\mathrm{iC1}+\mathrm{iC2}\right)\mathrm{R3}$ (3)

Rearranging (2) and substituting (3)

$\mathrm{iC1}+\mathrm{iC2}=\frac{\mathrm{Ein}}{\mathrm{R1}}-\frac{\mathrm{vC2}+\left(\mathrm{iC1}+\mathrm{iC2}\right)\mathrm{R3}}{\mathrm{R1}}$ Collecting terms:

$\mathrm{iC1}\left(1+\frac{\mathrm{R3}}{\mathrm{R1}}\right)+\mathrm{iC2}\left(1+\frac{\mathrm{R3}}{\mathrm{R1}}\right)=\frac{\mathrm{Ein}}{\mathrm{R1}}-\frac{\mathrm{vC2}}{\mathrm{R1}}$ (4) which is in the form (1)

From the schematic, $\mathrm{vC2}=\mathrm{iC1}\cdot\mathrm{R2}+\mathrm{vC1}$ or

$\mathrm{iC1}\cdot\mathrm{R2}=\mathrm{vC2}-\mathrm{vC1}$ (5) which is in the form (1).

The W array is formed from the LH sides of (4) and (5), the coefficients of iC1 and iC2:

$$W = \begin{bmatrix} 1+\dfrac{R3}{R1} & 1+\dfrac{R3}{R1} \\ R2 & 0 \end{bmatrix}$$

Array Q is formed from the RH sides, array S from Ein terms, and P from the reactive components:

$$Q = \begin{bmatrix} 0 & \dfrac{-1}{R1} \\ -1 & 1 \end{bmatrix} \qquad S = \begin{bmatrix} \dfrac{Ein}{R1} \\ 0 \end{bmatrix} \qquad P = \begin{bmatrix} C1 & 0 \\ 0 & C2 \end{bmatrix}$$

Then $C = (WP)^{-1}, \quad A = CQ, \quad B = CS$

The D and E arrays are obtained as follows:

It is not readily apparent from the schematic that one output expression for V1 is

$V1 = Vo1 = \dfrac{vC2 \cdot R1 + Ein \cdot R3}{R1 + R3}$. Then D and E are

$$D = \begin{bmatrix} 0 & \dfrac{R1}{R1+R3} \end{bmatrix}, \quad E = \frac{R3}{R1+R3}$$

However, it is easier to see from the schematic that another expression for V1 is the sum of the capacitor currents through R3 plus vC2 which is (3) above:

$$V1 = Vo2 = iC1 \cdot R3 + iC2 \cdot R3 + vC2$$

Here the new D and E arrays are:

$$D = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad E = 0$$

E = 0 since there are no Ein terms in Vo2.

The next step is to obtain the G = FP array. Array F is the coefficients of iC1 and iC2:

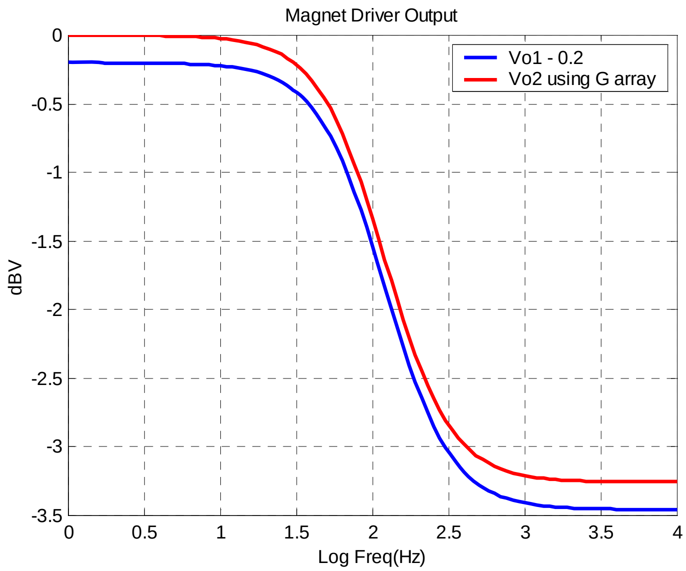
$F = \begin{bmatrix} R3 & R3 \end{bmatrix}, \quad G = FP$ where P is as given previously.

By substituting $\dfrac{dx}{dt} = Ax + B$ into $y = Dx + E + G\dfrac{dx}{dt}$ we get

$$y = (D + GA) \cdot (sI - A)^{-1} \cdot B + E + GB$$

See M-file `UsingGarray1.m` for output results.

The following plot is from that M-File:



The reader may well ask, how is the algebra "reduced" here? Conventional state-space analysis requires that each derivative term, (iC1 and iC2 here) be isolated on the LH side to create the standard form

$$\frac{dx}{dt} = Ax + Bu$$

(Recall that $iC1 = C1\frac{dVc1}{dt}$, $iC2 = C2\frac{dVc2}{dt}$)

Hence, without using the RA method, the algebra must continue as follows:

From (5)

$$iC1 = \frac{vC2}{R2} - \frac{vC1}{R2} \quad (5a)$$

From (4)

$$iC1 + iC2 = \left(\frac{Ein}{R1} - \frac{vC2}{R1}\right)\left(\frac{R1}{R1+R3}\right) \quad (4a)$$

$$iC1 + iC2 = \frac{Ein}{R1+R3} - \frac{vC2}{R1+R3} \quad (4b)$$

$$iC2 = \frac{Ein}{R1+R3} - \frac{vC2}{R1+R3} - iC1 = \frac{Ein}{R1+R3} - \frac{vC2}{R1+R3} - \frac{vC2}{R2} + \frac{vC1}{R2} \quad (4c)$$

$$iC2 = \frac{Ein}{R1+R3} + \frac{vC1}{R2} - vC2\left(\frac{1}{R1+R3} + \frac{1}{R2}\right) \quad (4d)$$

Eqns (5a) and (4d) are in the standard form. Thus

$$\begin{bmatrix} \dfrac{dVc1}{dt} \\ \dfrac{dVc2}{dt} \end{bmatrix} = \begin{bmatrix} \dfrac{-1}{R2 \cdot C1} & \dfrac{1}{R2 \cdot C1} \\ \dfrac{1}{R2 \cdot C2} & \dfrac{1}{C2}\left(\dfrac{1}{R1+R3}+\dfrac{1}{R2}\right) \end{bmatrix} \begin{bmatrix} vC1 \\ vC2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{C2(R1+R3)} \end{bmatrix} Ein$$

in which u = Ein, etc. The D and E arrays are obtained as before.

As N, the number of C's and L's increase, the algebra gets not only worse, but much worse. Hence the "reduced algebra" of forming the W, Q, and S arrays without having to continue to the standard form.

From M-File `RAconvention.m` verification file:

```
RA Method

A =
       -10000        10000
     2.5e+006 -2.6667e+006
B =
            0
  1.6667e+005

Conventional method

A =
       -10000        10000
     2.5e+006  2.6667e+006
B =
            0
  1.6667e+005
»
```

[1] *State Space Averaging with a Pocket Calculator*, R. Boyd, High Frequency Power Conversion Conference Proceedings, Santa Clara, CA, 1990, p.283