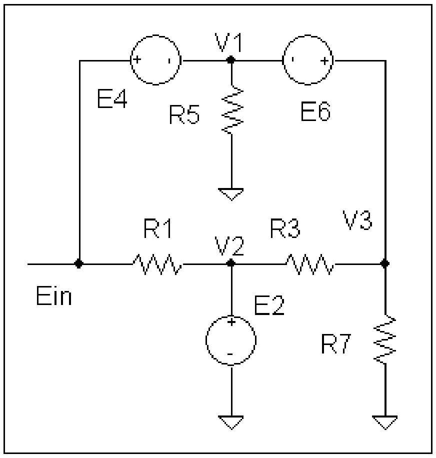
DS Method – Twin-T Network (60 Hz Notch Filter)

The converted circuit with 1V sources replacing the capacitors is



By noting that V2 = E2 = 1V, one unknown node is eliminated. Hence N = 3, U = 2, M =1. There are now two unknown nodes, V1 & V3. The output is taken from the second unknown node V3, hence Y = 2.

Component values are given in the M-file following. We need U+N = 5 dc equations. At node V1 we use KCL to get

$$i_{C4} + i_{C6} = \frac{V1}{R5}, \quad \text{or} \quad \frac{V1}{R5} - i_{C4} - i_{C6} = 0$$ placing unknowns on LH side.

At node V2 = E2, KCL gives

$$\frac{Ein - E2}{R1} = i_{C2} + \frac{E2 - V3}{R3} \quad \text{or} \quad i_{C2} - \frac{V3}{R3} = \frac{Ein}{R1} - E2\left(\frac{1}{R1} + \frac{1}{R3}\right)$$ knowns on RH side.

At node V3, KCL gives

$$\frac{E2 - V3}{R3} = \frac{V3}{R7} + i_{C6} \quad \text{or} \quad V3\left(\frac{1}{R3} + \frac{1}{R7}\right) + i_{C6} = \frac{E2}{R3}$$

Using KVL for the two capacitors (E4 and E6)

$$V1 = Ein - E4, \quad V3 - V1 = E6$$

The B1 array is skipped this time. The A1 array below contains the coefficients of the LH sides of the foregoing equations, and the B2 array contains the coefficients of the RH sides.

$$
\begin{array}{c|ccccc}
 & V1 & V3 & iC2 & iC4 & iC6 \\
\hline
A1 = & \multicolumn{5}{c}{\begin{bmatrix} \frac{1}{R5} & 0 & 0 & -1 & -1 \\ 0 & \frac{-1}{R3} & 1 & 0 & 0 \\ 0 & \frac{1}{R3}+\frac{1}{R7} & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \end{bmatrix}}
\end{array}
\qquad
\begin{array}{c|cccc}
 & E2 & E4 & E6 & Ein \\
\hline
B2 = & \multicolumn{4}{c}{\begin{bmatrix} 0 & 0 & 0 & 0 \\ -E2\left(\frac{1}{R1}+\frac{1}{R3}\right) & 0 & 0 & \frac{Ein}{R1} \\ \frac{E2}{R3} & 0 & 0 & 0 \\ 0 & -E4 & 0 & Ein \\ 0 & 0 & E6 & 0 \end{bmatrix}}
\end{array}
$$

Note again that every row and column of A1 and every column of B2 must have at least one non-zero entry.

Following the sequence of E2, E4, and E6 in B2 above,

$$
P = \begin{bmatrix} C2 & 0 & 0 \\ 0 & C4 & 0 \\ 0 & 0 & C6 \end{bmatrix}
$$

Again the rest uses template equations as shown in the M-file TwinT.m.

Introduction to Transient Analysis

Transient or time domain analysis is based on $\frac{\Delta x}{\Delta t} = Ax + Bu$ or $\Delta x = (Ax + Bu)\Delta t$

We already have the A & B arrays and we need to find the correct value of Δt. If Δt is too large the solution will not converge. If too small, excessive execution time may be required.

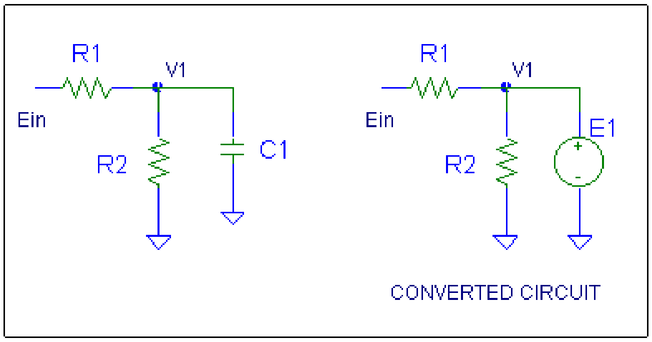
The time increment Δt generally should be smaller than the shortest time constant in A. Although this is not strictly true for every circuit and should only be used as a guide in selecting Δt. In MATLAB parlance

Tx=1/max(max(abs(A))) Shortest time constant in matrix A.

The time increment Δt is initially set at Tx/10 and then the mantissa is rounded down to an integer value. For example, Tx = 8.99e-4, then Δt = 8e-5 = 80us. Some adjustment of Δt may be required for circuits with very fast or very slow time constants.

Background information for the Time Response section of the M-file TwinT.m is given below:

A simple 1st order RC network is used as an example.



DC equations for the converted circuit:

$$\frac{\mathrm{Ein}-\mathrm{V1}}{\mathrm{R1}}=\frac{\mathrm{V1}}{\mathrm{R2}}+i_{C1}, \quad \text{or} \quad \mathrm{V1}\left(\frac{1}{\mathrm{R1}}+\frac{1}{\mathrm{R2}}\right)+i_{C1}=\frac{\mathrm{Ein}}{\mathrm{R1}}$$

And obviously V1 = E1. Forming A1, B2 and P as before:

$$\mathrm{A1}=\begin{bmatrix}\frac{1}{\mathrm{R1}}+\frac{1}{\mathrm{R2}} & 1\\ 1 & 0\end{bmatrix}, \qquad \mathrm{B2}=\begin{bmatrix}0 & \frac{\mathrm{Ein}}{\mathrm{R1}}\\ \mathrm{E1} & 0\end{bmatrix}, \qquad \mathrm{P}=\mathrm{C1}$$

Using values of R1 = 20K, R2 = 40K, and C1 = 0.5uF, calculating V, H, AB, etc., leads to

$$A=-150, \qquad B=100 \qquad D=1 \qquad E=0$$

A is dimension {N N} = {1 1} as expected.

Since $\left|\frac{1}{A}\right|=6.667\cdot\mathrm{ms}$, choose $\Delta t=500\cdot\mathrm{us}$.

We can now change Ein from a 1V input to a 4.5V step input and initialize the capacitor voltage Vc1 to zero. Vc1 can be set to any initial condition, but zero is usually chosen.

Using the form $\Delta x=(Ax+Bu)\Delta t$ we have

$$\Delta Vc_1=(A\cdot Vc_1+B\cdot \mathrm{Ein})\Delta t=0.225$$
$$Vc_2=\Delta Vc_1+Vc_1=0.225$$

$$\Delta Vc_2=(A\cdot Vc_2+B\cdot \mathrm{Ein})\Delta t=0.208$$
$$Vc_3=\Delta Vc_2+Vc_2=0.433$$

Combining:

$$Vc_2 = \left(A \cdot Vc_1 + B \cdot Ein\right)\Delta t + Vc_1 = 0.225$$
$$Vc_3 = \left(A \cdot Vc_2 + B \cdot Ein\right)\Delta t + Vc_2 = 0.433$$

Or in general, and setting the maximum number of iterations kmax = 100, and indexing k = 2,…,kmax

$$Vc_k = \left(A \cdot Vc_{k-1} + B \cdot Ein\right)\Delta t + Vc_{k-1}, \quad t = k \cdot \Delta t$$

The total time sweep or period is then $T = k\max \cdot \Delta t = 50 \cdot ms$

The output equation y = Dx + Eu is

$$Vo_k = D \cdot Vc_k + E \cdot Ein = Vc_k$$

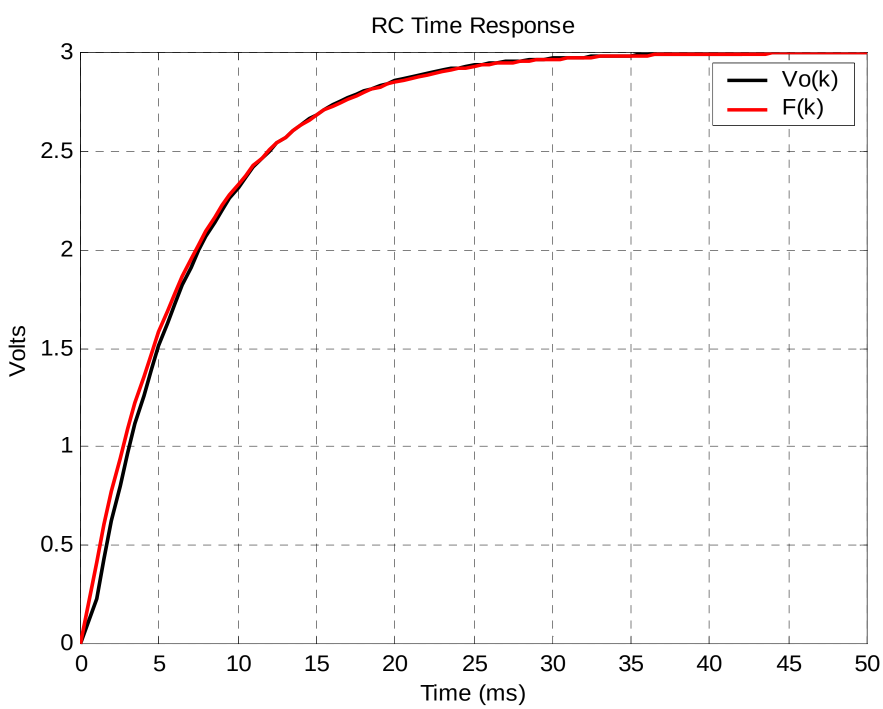
The circuit time constant is

$$\tau = Rp \cdot C1 = \left|\frac{1}{A}\right| = 6.667 \cdot ms, \quad 5\tau = 33.33 \cdot ms, \quad Rp = \frac{R1 \cdot R2}{R1 + R2}$$

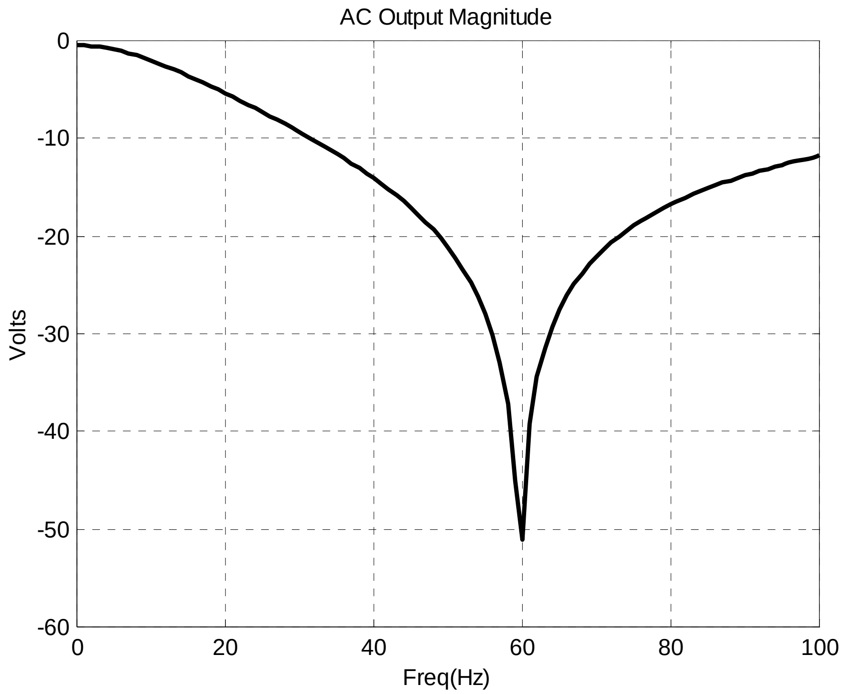
Note that the steady state dc output is $Vdc = \dfrac{Ein \cdot R2}{R1 + R2} = 3V$, as shown on the time plot.

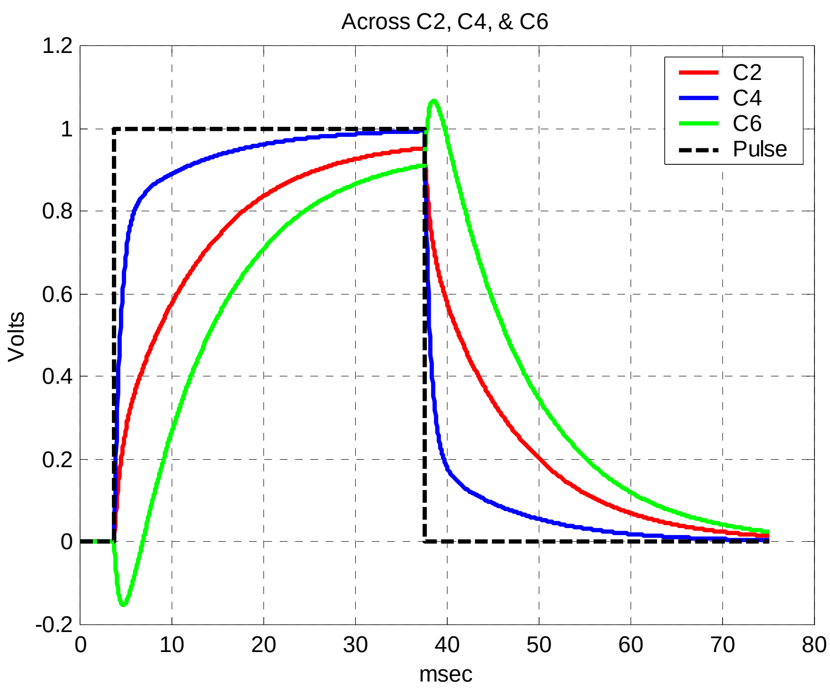
The output Vo(k) and $F(t) = \dfrac{Ein \cdot Rp}{R1}\left(1 - \exp\left(\dfrac{-t}{Rp \cdot C1}\right)\right)$ is plotted via the M-file rctime.m

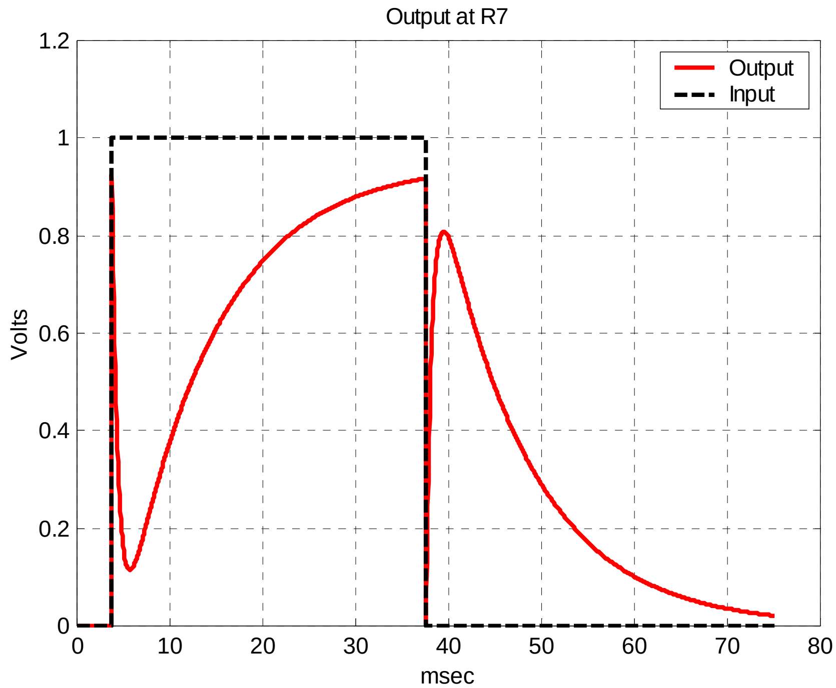
As will be seen, this method applies whether A, B, D, & E are scalars or arrays.



Output from TwinT.m







Output at R7
Output
Input
Volts
msec
1.2
1
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
70
80