Mathematical model and computational algorithm simulating the dynamic bahaviour of a mechanical element affected by load, active and load dependent friction forces (or torques)

The computing algorithm is intended to describe the dynamic behavior of a mechanical component or assembly characterized by mechanical ends of travel, under the action of dry friction passive forces (or torques), besides generic active. The mathematical algorithm is based upon the widely accepted hypothesis of Coulomb friction physical model, well suited to the dynamic simulation of motion transmissions of the mechanical systems, under external loads. According to the Coulomb friction hypothesis, the friction force or torque FF depends on the velocity DXJ as shown in figure 1, in which FD is the force (or torque) invariant with the speed and opposing it in dynamic (slipping) conditions and FS is the maximum absolute value which the friction force (or torque) can assume in static (sticking) conditions.

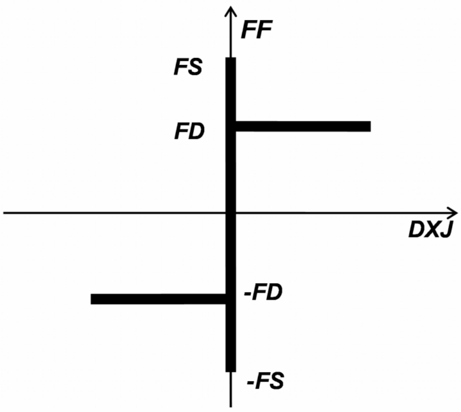

Figure 1

Figure 2 shows the schematic representing the position and the velocity of the moving element, the forces acting on it and the corresponding sign convention.

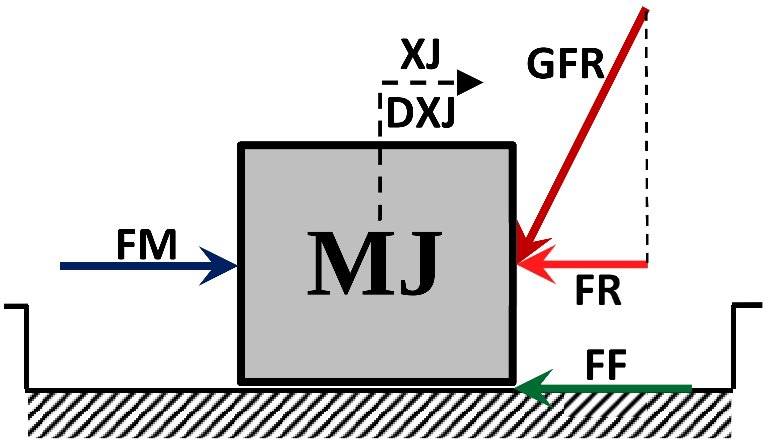

Figure 2

This model is able to take into account both the friction forces (or torques) independent and dependent on the external load FR, both in case of load opposing and aiding the motion; nevertheless, it considers the interaction between the mechanical element and its two eventual ends of travel. In fact, the proposed model considers the friction force FF as a sum of two terms: the former is the load invariant term (FDJ) and the latter is the load dependent one. The relationship between FR and the related friction term is obtained by means of the proper amount of efficiency characterizing the mechanical element. When FR acts as opposing the motion, the efficiency to be considered is EtO (expressed as the ratio between resisting and driving power) and the related friction force is obtained as (1/EtO-1)· |FR|; when FR is aiding the motion, the efficiency to be considered is EtA (defined as the inverse ratio with respect to EtO) and the related friction force is obtained as (1-EtA)· |FR|. The mentioned efficiencies are conventionally defined as above reported, in order to assign current values anyhow lower than 1, as it is usual.

So, the total friction force (or torque) in slipping condition is FD=FDJ+(1/EtO-1)·FR under an opposing load and FD=FDJ+(1-EtA)·FR under an aiding one.

It must be noted that the amount of EtO must lie in the interval between 0 and 1 in order to allow the motor drives the system, while the value of EtA must be not greater than 1; if the mechanical system is reversible the amount of EtA must lie in the interval between 0 and 1, if it is irreversible EtA must be not greater than 0 and the negative value is an expression of the irreversibility degree of the mechanical system. In fact, in slipping and aiding conditions, if EtA=0, the load produces a friction force FF that is equal and opposing to the load itself (FF=FR) and their net effect is null, so requiring no action by the motor (neither driving nor breaking); if EtA=-0.5 (or EtA=-1), FR develops an opposing amount of FF=1.5·FR (or FF=2·FR) and their net effect results in a force opposing the motion and amounting to 0.5·FR (or 1·FR), so requiring a driving action of 0.5·FR (or 1·FR) by the motor. All these conditions are properly simulated by the algorithm.

The model is conceived to compute the friction force (or torque) FF in slipping conditions; in case of sticking conditions, the maximum value which can be assumed by FF is obtained multiplying the slipping one by FSD (static to dynamic friction ratio equal or greater than 1). When the sticking condition persists, the absolute actual amount of FF – requested to balance the active force – is not greater than the above considered maximum value.

It must be noted that five possible conditions can occurs in each computational step:

a) Mechanical element initially sticking which must persist in sticking condition, being the absolute value of the active forces (and consequently of FF) not greater than FSD·FD;

b) Mechanical element initially sticking which must breakaway, so turning to slipping condition, being the absolute value of the active forces greater than FF=FSD·FD;

c) Mechanical element initially slipping which must keep the slipping condition in the same velocity sense (either when the absolute value of the active forces/torques is greater than FF=FD, or simply when the element velocity has no-sign reversion within the considered computational step under all the forces acting on it, with respect to its inertia);

d) Mechanical element initially slipping which must stop, so turning to sticking condition (having the velocity a potential sign reversion within the computational step, as a consequence of inertia and applied forces);

e) Mechanical element initially slipping which must keep the slipping condition, following a motion reversion within the computational step, having the active forces/torques a value greater than FF= FSD·FD and the sense opposing the initial motion.

The proposed dynamic simulation algorithm is able to distinguish among the conditions a), b), c) and d), solving them within the single considered computational step; the condition e) is performed by means of two following computational steps: the present step is considered as case d) and the following one as case b). It must be noted that this procedure computes a marginally time delayed breakaway (on an average the time-delay amounts to half computing interval).

All these abilities are performed in case of both opposing and aiding load with respect to the actual movement or the eventual break-away (incipient motion). It must be noted that, when the external load aids the breakaway, the friction force to be considered is obtained through EtA, while when the external load opposes the breakaway, the incipient motion is performed against a friction force amount depending on EtO; so the procedure, evaluating the eventual breakaway, constrains the FF amount within two different limits, ruled by EtO and EtA respectively besides by FSD (turning the slipping friction value into the sticking as previously said).

Among the surprising abilities of the algorithm (initially unexpected even by the authors) , the following is remarkable: for example, if the aiding efficiency value amounts to EtA=0.2, together with a static to dynamic friction ratio of FSD=1.5, the friction torque in slipping (dynamic) conditions, related to a load of 25000 N·m, has an actual value of 20000 N·m, so performing a reversible behavior, as previously described; on the contrary, the friction torque in sticking (static) conditions, referred to the same load, can reach a maximum value of 30000 N·m (in breakaway), so producing an irreversible behavior. In this case, the breakaway of the system (initially sticking) in case of aiding load requires the driving action of the motor element, followed by its braking action when the system turns to a slipping condition (particularly clear when the system sets its motion at a constant speed).

List of symbols

DXJ	movable element velocity
EtA	efficiency (ratio between resisting and driving power) when FR aids DXJ
EtO	efficiency (ratio between resisting and driving power) when FR opposes DXJ
FD	global friction force (or torque) in slipping conditions
FDJ	load independent friction force (or torque) in slipping conditions
FF	global friction force (or torque)
FM	driving force (or torque)
FR	load component along the element motion direction (parallel to DXJ)
FS	maximum value of global friction force (or torque) in sticking conditions
FSD	static (sticking) to dynamic (slipping) friction ratio
FSJ	maximum value of load independent friction force (or torque) in sticking conditions
GFR	global external load
XJ	movable element position