Development of a Scilab/Matlab Toolbox Dedicated to LTI/LPV Descriptor Systems for Fault Diagnosis ⋆

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Abstract: The paper discusses the development of a MATLAB/SCILAB toolbox for systems modeled in the LTI or LPV descriptor framework. The properties of regularity, solvability, controllability and observability are presented. Full and reduced order, proportional and proportional-integral observers are included. Some of these observers consider unknown inputs. The main contribution is provide a toolbox than can be used as auxiliary in state-estimation and fault detection based observers. These observers have been considered from few papers published recently. The fault detection and isolation can be achieved by the construction of bank of observers. These banks of observers can be built by the selection of the input/output matrices or automatically by using the proposed algorithms.

Keywords: Descriptor systems, singular systems, LPV, Fault diagnosis, GOS, DOS.

1. INTRODUCTION

Many systems can be modeled with nonlinear differential equations, however the design of monitoring and control systems are a difficult task for this kind of representation. For this reason, it is very common that nonlinear systems are linearized to obtain Linear Time Invariant (LTI) systems, but this representation is valid around one equilibrium or operation point.

One way to improve the model representation is to include some restrictions. If the restrictions are part of the model, then the system becomes into a descriptor-LTI (DLTI) representation. The main advantage of DLTI systems is the integration of static relationships (e.g. physical restraints) and dynamical relationships. These considerations allow to model a wide range of processes, e.g. electrical, mechanical and hydraulics applications were studied in (Dai, 1989; Duan, 2010).

Sufficient conditions for the existence of a Luenberger observer were given in (Hou and Muller, 1999; Darouach and Boutayeb, 1995). The authors in (Darouach et al., 1996) presented a reduced order unknown input observed similar to the observers for LTI systems that were studied in (Chen and Patton, 1999). A proportional-integral unknown input observer (PIUIO) was proposed in (Koenig and Mammar, 2002), and it was extended to fault detection application in (Koenig, 2005), applied to a three machine infinite bus system.

This paper presents a compilation of some methodologies for stability analysis, observer design and, fault detection on systems in descriptor form, from LTI extended to linear parameter varying (LPV). The main contribution is to propose a tool that can be used in the analysis of DLTI. Design full and reduced order observers with known and unknown inputs, e.g Luenberger, proportional with unknown inputs (P-UIO) and proportional-integral with unknown inputs (PI-UIO). Some of these observers were extended to the LPV case. For this case, some functions based on the gain scheduling approach are presented. The main difference of this toolbox as compared to others, for instance, the one in (Varga, 2000), is the presence of functions for observer desing and fault detection (FD).

This paper is organized as follows. Some definitions about descriptor systems are presented in Section II. The description of the toolbox and its main functions are presented in Section III. Some examples using the toolbox are given in Section IV for DLTI and Section V for DLPV. Finally some conclusions are discussed in section VI.

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2. DESCRIPTOR LINEAR TIME-INVARIANT SYSTEMS

Let us consider in a first step a descriptor linear time-invariant (DLTI) system defined as

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where A, $E \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $C \in \mathbb{R}^{p\times n}$ are known matrices with appropriate dimension. $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$ are the state vector, the control input and, the measured output respectively.

When unknown inputs are considered the system becomes

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t) + Rd(t) \\ y(t) &= Cx(t) \end{aligned} \tag{2}$$

where $R \in \mathbb{R}^{n\times r}$ is a known matrix of appropriate dimension and $d(t) \in \mathbb{R}^r$ the unknown input vector.

The main difference among LTI and descriptor LTI (DLTI) systems, is that matrix E of (2) is singular and therefore non-invertible. Two of the most important properties are the solvability and the regularity. If these properties hold, a singular system can be converted to Kronecker-Weistrauss restricted system equivalence form (r.s.e). In this form some properties like controllability, observability and stability are easier to determine.

The regularity is related to the invertibility of the pair pencil (E, A). In (Gantmacher, 1959) the regularity is defined as

Definition 1. The system (2) is called regular if there exists a scalar γ such that $\det(\gamma E - A) \neq 0$ or equivalently, the polynomial $\det(sE - A)$ is not identically zero. In this case the matrix pair (E, A), or the matrix pencil $sE - A$ is regular.

Solvability is defined as the existence of a unique solution for any given sufficiently differentiable $u(t)$ and any given admissible initial conditions corresponding to the given $u(t)$.

Definition 2. (Yip and Sincovec, 1981) The matricial relationship (E, A) is solvable if and only if $|sE - A| \neq 0$, or equivalently if there exists an scalar $\lambda \in \mathbb{C}$ such that $|\lambda E - A| \neq 0$.

If the system is regular, then the following hold:

Definition 3. (Sokolov, 2006) Two systems (E, A, B, C) and $(\hat{E}\,\hat{A}, \hat{B}, \hat{C})$ are defined as equivalent, or restricted systems equivalent (r.s.e.), if their order, number of inputs and outputs are equal and there exist two non singular matrices P and Q such that $\hat{E} = PEQ$, $\hat{A} = PAQ$, $\hat{B} = PB$, $\hat{B} = CQ$. Where

$$PEQ = \begin{bmatrix} I_n & 0 \\ 0 & N \end{bmatrix} \tag{3}$$

$$PAQ = \begin{bmatrix} A_1 & 0 \\ 0 & I_r \end{bmatrix} \tag{4}$$

when $A_1 \in \mathbb{R}^{n_1\times n_1}$, $I_n \in \mathbb{R}^{n\times n}$ is the identity matrix and N is a nilpotent matrix of index r such that $N^r = 0$ but $N^{r-1} \neq 0$. The equivalent system is:

$$\left.\begin{aligned} \dot{x_1} &= A_1x_1(t) + B_1u(t) \\ y_1(t) &= C_1x_1(t) \end{aligned}\right\} \text{Slow subsystem} \tag{5}$$

$$\left.\begin{aligned} N\dot{x_2} &= x_2(t) + B_2u(t) \\ y_2(t) &= C_2x_2(t) \end{aligned}\right\} \text{Fast subsystem} \tag{6}$$

or equivalently

$$\begin{bmatrix} I & 0 \\ 0 & N \end{bmatrix}\begin{bmatrix} \dot{x_1}(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \tag{7}$$

$$y(t) = [\, C_1 \;\, C_2 \,]\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

with

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = PB, \quad [C_1\, C_2] = CQ$$

Subsystem (5) represents the *slow response* and subsystem (6) represents the *fast response*. The system (7) also is called the Kronecker-Weistrauss canonical form.

However, if the system is not regular but

$$\mathsf{rank}\begin{bmatrix} E & A \\ 0 & E \\ 0 & C \end{bmatrix} = n + \mathsf{rank}\, E \tag{8}$$

Then a equivalent form based on QR transformation (Darouach and Boutayeb, 1995) can be obtained as

$$PE = \begin{bmatrix} \bar{E} \\ 0 \end{bmatrix} \quad PA = \begin{bmatrix} \bar{A} \\ \bar{A}_1 \end{bmatrix} \quad PB = \begin{bmatrix} \bar{B} \\ \bar{B}_1 \end{bmatrix} \tag{9}$$

where $\begin{bmatrix} \bar{E} \\ 0 \end{bmatrix}$ is found applying QR decomposition.

3. DESCRIPTOR SYSTEM PACKAGE

Due to the lack of special tools for the analysis and observer design for descriptor LTI and LPV systems, It was necessary program some scripts, in order to solve the problem. This functions were developed for SCILAB and MATLAB platforms. The SCILAB version can be downloaded from ATOMS web page (López-Estrada et al., 2011). The MATLAB version is under revisions.

It should be noted that currently there are few works about it. For example, in (Varga, 2000) a toolbox for MATLAB is presented which is focused principality on the solution of numerical problems. The functions programmed in the toolbox can be used for i) analysis, ii) observer design and iii) Fault detection for DLTI and LPV systems. These functions are summarized in Table 1.

Some restrictions are listed below

- As seen in Table 1, the descriptor package can be used like an auxiliary tool in the analysis and observer design for DLTI and DLPV. Some of the observers were used to built bank of observers using a generalized observers schemes (GOS) or dedicated observers schemes (DOS) for DLTI and DLPV(Frank, 1990). However, the algorithms are limited only to compute the gains of each one of the observers.

Table 1. Descriptor systems toolbox

Function	Description
	Analysis of properties
dss2tf	DLTI to transfer function
dcontr	C, R and I controllability matrices
dobsv	C, R and I observability matrices
dstabil	Stability
qrrse	r.s.e based in QR decomposition
invrse	Inverse r.s.e
kwrse	Kronecker-Weistrauss r.s.e† ⋆
dc2dd	Continuous to discrete-time ⋆
fundmatrix	Laurent expansion⋆
	Observers design
darobsv95	Full order observer†
redobsv95	Reduced order observer †
darobsv96	Reduced order observer with unknown inputs†
puiobsv	Proportional UIO (P-UIO)
piuiobsv	Proportional-integral UIO (PIUIO)
LPVpuiobsv	P-UIO for DLPV systems
LPVpiuiobsv	PI-UIO for DLPV systems
	Fault detection for DLTI and DLPV
gosbank1/2	GOS bank using a P-UIO and PIUIO
dosbank1/2	DOS bank using a P-UIO and PIUIO
lpvgosbank1	GOS bank using a P-UIO
Others	
lpvweig2/3/4	Weighting functions of 2,3 or 4 vertex
	†Only for DLTI
	⋆Only for SCILAB

- Some functions are only available for SCILAB, due that some requirements are not achievable in MATLAB.
- The *YALMIP* toolbox (Lofberg, 2004) is necessary to solve some linear matrix inequalities (LMI) in the MATLAB package. By default the solver *lmilab* is selected, however this can be changed by *sedumi* or other.

4. NUMERICAL EXAMPLE

This section shows some examples of how to work with the descriptor systems toolbox. Consider the following descriptor system

$$E = diag[1,1,0,0]$$

$$E\dot{x}(t) = \underbrace{\begin{pmatrix} -0.7 & 1 & 0 & 0 \\ -1 & -0.8 & 0 & 0 \\ 0 & -1 & -0.7 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}}_{A} x(t) + \underbrace{\begin{pmatrix} 1 & 10 \\ 1 & 0.5 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}}_{B} u(t) + \underbrace{\begin{pmatrix} 1 \\ 0.5 \\ 0 \\ 0 \end{pmatrix}}_{R} d(t)$$

$$y(t) = \underbrace{\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}}_{C} x(t)$$

In MATLAB the descriptor system is defined as

```
E=diag([1,1,0,0]);
A=[-0.75 1 0 0 ; -1 -0.85 0 0;
   0 -1 -0.75 0; 0 0 0 -1];
B=[1 10; 1 0.5 ; 0 0; 1 0];
R=[1 0. 0 0]';
C=[1 0 1 0; 0 1 0 1; 1 0 1 1];
```

```
dsys=dsystem (E,A,B,C,R)
```

4.1 Equivalent systems

The Kronecker-Weistrauss canonical form (7) is computed as

```
 [A1, N, h, B1, B2, C1, C2]=kwrse  (E,A,B,C)
C2  =

 - 1.1547005    0.
   0.           1.
 - 1.1547005    1.
C1  =

   1.0327956  - 1.
 - 0.7745967    0.
   1.0327956  - 1.
B2  =

   0.    0.
 - 1.    0.
B1  =

 - 1.2909944  - 0.6454972
 - 1.         - 10.
h  =

   1.
N  =

   0.    0.
   0.    0.
A1  =

 - 0.85       - 1.2909944
   0.7745967  - 0.75
```

A restricted system equivalent system based on QR transformation (9) is determined as

```
  [Ee, Ae, Ae1, Be, Be1, Ce]=qrrse(E,A,B,C)
Ee =
     1      0      0      0
     0      1      0      0
Ae =
   -0.75    1.00          0          0
   -1.00   -0.850         0          0
Ae1 =
          0   -1.00   -0.75     0
          0    0           0   -1.00
Be =
    1.00   10.00
    1.00   0.50
Be1 =
     0      0
     1      0
Ce =
    0   -1.00 -0.75  0
    0    0     0    -1.00
    1.00 0     1.00  0
    0    1.00  0     1.00
    1.00  0    1.00  1.00
```

4.2 Observability and controllability

In descriptor systems there exists principally three types of controllability/observability: Complete, Reachable and

Impulsive. For the case of C-controllability, it is related with the fact that the system can be controlled with any initial conditions Duan (2010). R-controllability means that the system only can be controlled by a reduced set of initial conditions and I-Controllability means that impulsive signals can be controlled.

The controllability in descriptor systems can be found in different ways, e.g. using Laurent expansion, equivalent systems or from direct approaches. In the Laurent expansion approach, the controllability is related with the dimension of (E, A) and the rank of the Laurent expansion matrices. In the restricted system approach the controllability can be found by the analysis of controllability of slow (5) and fast (6) subsystems (Duan, 2010). While in the direct form approach the controllability is found by analyzing matrices (E, A, B) (see (Yip and Sincovec, 1981)). Similar definitions are considered for the case of observability.

The observability condition can be tested by the following command

```
[nC,Co_mat,nR,Ro_mat,nI,Io_mat]=dobsv(E,A,C)
nI =
    Iobsv: 'T'
nC =
     4
Co_mat =
    SlowS: [7x4 double]
    FastS: [7x4 double]
nR =
    Obsv: 'T'
    Rank: 4
Ro_mat =
    1.7500   -1.0000         0         0
    1.0000    1.8500         0         0
         0    1.0000    0.7500         0
         0         0         0    1.0000
    1.0000         0    1.0000         0
         0    1.0000         0    1.0000
    1.0000         0    1.0000    1.0000
nI =
    Iobsv: 'T'
     rank: 6
```

The results shows that the system is C,R,I-observable.

The controllability can be determined as

```
  [Ccon, Rcon, Icon]=dcontr (E,A,B)
Ccon =
      Des: 'The system is not C-Controllable'
    Ranks: [4 3]
     Fast: [4x6 double]
     Slow: [4x6 double]
Rcon =
     Des: 'The system is R-Controllable'
    Rank: 4
     Mat: [4x6 double]
Icon =
     Des: 'The system is I-Controllable '
    Rank: 6
     Mat: [8x10 double]
```

4.3 Observer design

The observation of the system behavior, in continuous-time, allows us to gain indirect information on the appearance of the unknown signals (Trumpf, 2007). These signals can be estimated by an unknown input observer. An observer is defined as an unknown input observer (UIO), if its state estimated error vector $e(t)$ tends to zero asymptotically, regardless of the presence of the unknown input in the system. A popular observer that has been widely studied is the observer proposed in (Darouach and Boutayeb, 1995). This observer has been extended to fault diagnosis in LTI and LPV systems in (Koenig, 2005; Hamdi et al., 2009). The observer is based in a Luenberger form as

$$\dot{z}(t) = Nz(t) + L_1u(t) + L_2y(t) + Gu(t) \qquad (10)$$
$$\hat{x}(t) = z(t) + by(t) + Kd\,y(t) \qquad (11)$$

where N, L_1, L_2 and K are matrices of appropriate dimension. b and d are vectors obtained from

$$a\bar{E} + b\bar{C} = I_n \qquad (12)$$
$$c\bar{E} + d\bar{C} = 0 \qquad (13)$$

The observer gains can be computed as

```
poles=[-4,-3,-2 -1]
 [N,K,L1,L2,G,test]=darobsv95(dsys,poles)
  nI =
    Iobsv: 'T'
K =
    0.45   -2.84    1.01
    0.63   -0.29    0.57
   -0.27    0.88   -0.42
   -0.05   -0.04   -0.03
L1 =
   -3.36    3.27    0.37    6.17   -2.89
   -0.75   -0.68   -0.62   -0.75    0.06
    1.54   -0.56   -0.70   -0.42   -0.13
   -0.12    0.46    0.18   -0.25   -0.27
L2 =
    1.78    0.53    0.93    0.12    0.40
   -1.14    0.22   -0.30    0.81   -0.55
   -0.93    0.08    0.69   -0.51    0.60
    0.27   -0.39   -0.09    0.30    0.30
G =
    2.75    4.87
    0.81   -1.37
   -1.40   -4.71
   -0.22   -0.98
```

4.4 Fault detection and isolation based observers

The fault detection can be done with banks of observers by using a generalized observer scheme (GOS) or dedicated observer scheme (DOS) (Frank, 1990). Considering the proportional unknown input observer (PUIO) proposed in (Hamdi et al., 2009)

$$\dot{z}(t) = Nz(t) + G_1u(t) + Ly_o(t) \qquad (14)$$
$$\hat{x}(t) = z(t) + H_2y_o(t) \qquad (15)$$

For the numerical example, if the observer is stabilized in a LMI region "$\alpha = -2$," (see (Hamdi et al., 2009)) then the gains for each one of the observers in the GOS bank (for sensor) are computed as

```
 >> lmizone=-10
%'s'= sensors
[N,G1,L,H2,Bo1]=gosbank1(dsys,lmizone,'s')
C =
```

```
     0     1     0     1
     1     0     0     0
C =
     1     0     1     0
     1     0     0     0
C =
     1     0     1     0
     0     1     0     1
N(:,:,1) =
   -0.0001    0.0000    0.0000    0.0000
   -0.6667   -0.5668   -0.0001   -0.0001
    0.8889    0.7555   -0.0000    0.0000
    0.3333    0.2833    0.0000   -0.0001
N(:,:,2) =
   -0.2401   -0.2040   -0.0000         0
   -0.6800   -0.5780   -0.0000         0
    0.4800    0.4080   -0.0000         0
         0         0         0   -0.0000
N(:,:,3) =
   -0.8889   -0.7556   -0.0000    0.0000
   -0.6667   -0.5667   -0.0000   -0.0001
    0.8888    0.7555   -0.0001    0.0000
    0.3333    0.2833    0.0000   -0.0001
G1(:,:,1) =
         0         0
    0.6667         0
   -0.8889         0
   -0.3333         0
G1(:,:,2) =
    0.2400         0
    0.6800         0
   -0.4800         0
         0         0
G1(:,:,3) =
    0.8889         0
    0.6667         0
   -0.8889         0
   -0.3333         0
L(:,:,1) =
         0   -0.0000   -0.0000    0.0000
    0.0000   -0.1889   -0.1889   -0.6667
   -0.0000    0.2519    0.2519    0.8889
   -0.0000    0.0944    0.0944    0.3333
L(:,:,2) =
    0.0077         0    0.0058   -0.2458
    0.0218         0    0.0163   -0.6963
   -0.0154         0   -0.0115    0.4915
         0         0         0         0
L(:,:,3) =
   -1.1852   -0.6469   -0.8889   -0.6469
   -0.8889   -0.4852   -0.6667   -0.4852
    1.1852    0.6469    0.8889    0.6469
    0.4444    0.2426    0.3333    0.2426
H2(:,:,1) =
         0         0         0    1.0000
   -0.0000    0.3333    0.3333         0
   -1.3333   -0.4444   -0.4444         0
    0.0000   -0.6667    0.3333         0
H2(:,:,2) =
    0.2400         0    0.1800    0.8200
   -0.3200         0   -0.2400    0.2400
   -0.4800         0    0.6400   -0.6400
         0   -1.0000         0         0
H2(:,:,3) =
    1.3333    0.4444    1.0000    0.4444
   -0.0000    0.3333   -0.0000    0.3333
   -1.3333   -0.4444    0.0000   -0.4444
    0.0000   -0.6667    0.0000    0.3333
Bo1(:,:,1) =
     0     0
     1     0
Bo1(:,:,2) =
     0     0
     1     0
Bo1(:,:,3) =
     0     0
     1     0
```

where $(:,:,j)$, with $j = 1, 2, 3$, indicates the number of the observer. The function selects automatically the output matrices.

5. DISCRETIZATION

Considering a regular descriptor system (1), the Laurent expansion of $(sE - A)^{-1}$ is (Karampetakis, 2003)

$$\Phi(s) = (sE - A)^{-1} = \Phi_{-k}s^{k-1} + \cdots + \Phi_{-2}s + \Phi_0 s^0 + \Phi_1 s^{-1} + \ldots \Phi_k s^{-k-1} \tag{16}$$

$$= \sum_{k=h}^{\infty} \Phi_k \left(E, A\right) s^{-k-1} \tag{17}$$

where $k = 1, 2, ...h$, h is the nilpotence index of $(sE - A)$ (Lewis, 1985, 1990), and Φ_k are the fundamental matrices defined by the Drazin inverse of the matrix A (Bernstein, 2009; Jun, 2002). Using the Kronecker-Weistrauss transformation the matrices Φ_0 and Φ_{-1} can be defined as

$$\Phi_0 = P \begin{bmatrix} A_1^0 & 0 \\ 0 & 0 \end{bmatrix} Q \tag{18}$$

$$\Phi_{-1} = P \begin{bmatrix} 0 & 0 \\ 0 & -N^0 \end{bmatrix} Q \tag{19}$$

Then, using the Laurent coefficients, the continuous system (2), can be rewritten as (Karampetakis, 2003):

$$\underbrace{\begin{bmatrix} \rho I_n - \Phi_0 A & 0 \\ 0 & I_n + \rho \Phi_{-1} E \end{bmatrix}}_{\rho\tilde{E} - A} \underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}}_{\tilde{x}(t)} = \underbrace{\begin{bmatrix} \Phi_0 B \\ \Phi_{-1} B \end{bmatrix}}_{\tilde{B}} u(t) \tag{20}$$

The system (20) can be discretized by the following theorem:

Theorem 4. (Karampetakis, 2003) Using a zero-order hold approximation of the input $u(t)$ and first-order hold approximation of the derivatives of the input $u(t)$, the continuous time non-homogeneous singular system is discretized as

$$\left\{ \begin{array}{rl} x_1((k+1)T) & = \tilde{A} x_1(kT) + \tilde{B}_1 u(kT) \\ \tilde{E}_1 x_2((k+1)T) & = x_2(kT) + \tilde{B}_2 u(kT) \end{array} \right\} \tag{21}$$

$$x(kT) = \left[\, I_n \;\; I_n \,\right] \begin{bmatrix} x_1(kT) \\ x_2(kT) \end{bmatrix}$$

where:

$$\tilde{A} = e^{\Phi_o AT} \tag{22}$$

$$\tilde{B}_1 = \left[\int_0^T e^{\Phi_0 A\tau} d\tau\right] \Phi_0 B \tag{23}$$

$$\tilde{E}_1 = (\Phi_{-1}E - T \times I_n)^{-1}\Phi_{-1}E \tag{24}$$

$$\tilde{B}_2 = T(\Phi_{-1}E - T \times I_n)^{-1}\Phi_{-1}B \tag{25}$$

Based in this, the next algorithm is proposed to find the corresponding discrete system of (1):

Step 1 Check if the pair (E, A) is regular, if $\mathsf{det}(\gamma E - A) \neq 0$, then continue.
Step 2 Find the matrices (P, Q) of r.s.e form
Step 3 Compute the Drazin inverse, then find Φ_0 and Φ_{-1}
Step 4 Compute $\Phi_{-k} = -\Phi_{-1}E\Phi_{-k+1} = \Phi_{-1}\left(-E\Phi_{-1}\right)^k$, for $k = 2, 3, ..., h$ and $\Phi_k = \Phi_0\left(A\Phi_0\right)^k = \Phi_0 A\Phi_{k-1}$, $k = 1, 2, ...$
Step 5 Compute $\tilde{A}$, $\tilde{B}_1$, $\tilde{E}_1$, $\tilde{B}_2$ from (22, 23, 24) and (25) respectively.
Step 6 Rewrite equations in the form of (21)

For the example given above, considering a sample time $T_s = 0.1$ the discrete system can be computed as

```
[E1t,Ad, B1t,B2t]=c2dd (E,A,B,0.1)
 B2t  =

    0.    0.
    0.    0.
    0.    0.
  - 1.    0.
 B1t  =

    0.1007641     0.9638660
    0.0910875     0.0017686
  - 0.12145     - 0.0023581
    0.            0.
 Ad  =

    0.9231240     0.0921582    0.    0.
  - 0.0921582     0.9139082    0.    0.
    0.1228777     0.1147890    1.    0.
    0.            0.           0.    1.
 E1t  =

    0.    0.    0.    0.
    0.    0.    0.    0.
    0.    0.    0.    0.
    0.    0.    0.    0.
```

6. DESCRIPTOR LPV SYSTEMS

Continuous descriptor LPV systems (DLPV), in polytopic form, under unknown inputs (noise, disturbances, etc) are usually defined as

$$E\dot{x}(t) = \sum_{i=1}^{h} \rho_i(\theta(t)) \left\{A_i x(t) + B_i u(t) + R_i d(t)\right\} \tag{26}$$

$$y(t) = Cx(t) \tag{27}$$

where $A_i, E \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ and $R_i \in \mathbb{R}^r$ are known matrices with appropriate dimension. $x(t)$, $u(t)$, $y(t)$ and $d(t)$ are the state vector, the control input, the measured output and the unknown input vector respectively. The parameter $\theta(t)$ is assumed to be bounded and lies into a hypercube such that

$$\theta(t) \in \Gamma = \left\{\theta | \underline{\theta}_i(t) \leq \theta(t) \leq \bar{\theta}_i(t)\right\}, \ \ \forall t > 0 \tag{28}$$

The functions $\rho_i\theta(t)$ are the weighting function that permits the commutation among the models. The polytopic DLPV system is scheduled through the following convex set

$$\forall i \in \{1, 2..., h\}, \ \rho_i(\theta(t)) \geq 0, \ \sum_{i=1}^{h} \rho_i(\theta(t)) = 1 \tag{29}$$

6.1 Numerical example

Consider the numerical example presented in (Hamdi et al., 2012), the gains of the PIUIO can be computed as

```
    %% descriptor-LPV system
  dsys=dsystem(E,Ai,Bi,C,Ri)
  alpha=3; %LMI zone

  %  observer gains
[N,G,L,Ti,H2,Phi]=lpvpiuiobsv(dsys,alpha)
```

where $(A,\ B,\ R\,)$ are in polytopic form $\forall i \in \{1, 2, ..h\}$ and, alpha (α) is to define a zone in the left part of the complex plan bounded with a line of abscissa ($-\alpha$) where $\alpha \in \mathbb{R}^+$.

The resulting matrices are not presented here because the sizes are very large. However, others observers can be achieved similar to the examples in Sections 4.3 and 4.4 (see the Table 1).

7. CONCLUSIONS

This paper presents a set of functions that have been written in order to develop a descriptor systems toolbox for SCILAB and MATLAB. The toolbox contains a set of tools dedicated to analyze descriptor systems, from LTI extended to LPV. These tools include the implementation of algorithms for the analysis of controllability, observability, stability and to find the fundamental matrices of the Laurent expansion. Some functions to design state-observers are proposed. Some of these observers are considered for fault detection schemes, for the construction of manual or automatic banks of observers. The toolbox can also be used for discretization of continuous systems by the method proposed in (Karampetakis, 2003) (see Table 1).

It is very important to note that this paper is not a survey and therefore, the examples presented here are only to demonstrate the usefulness of the toolbox. Therefore, we recommend a detailed review of the references.

The SCILAB version of the toolbox is available to the community in the SCILAB files exchange web page (López-Estrada et al., 2011). The toolbox was created on the basis of free software, so anyone can improve it and modify it freely. The MATLAB package is under revisions and can be requested by email to the authors.

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