MATLAB Exercise – Single Pole Cepstrum

Program Directory: `matlab_gui\single_pole_cepstrum`
Program Name: `single_pole_cepstrum_GUI25.m`
GUI data file: `single_pole_cepstrum.mat`
Callbacks file: `Callbacks_single_pole_cepstrum_GUI25.m`
TADSP: Example 8.1, p. 413, Section 8.43, pp. 438-440, Problem 8.11

This MATLAB exercise compares and contrasts three methods for computing the complex cepstrum of a signal whose $z-$transform consists of a single pole system with pole radius less than one (i.e., real root inside the unit circle). The three computation methods for computing the complex cepstrum of this minimum-phase signal are the analytical method, the recursion method for a minimum-phase signal, and the conventional method based on computing the inverse Fourier transform of the complex logarithm of the Fourier transform of the signal.

Single Pole Cepstrum – Theory of Operation

The single pole, input signal impulse response is computed as:

$$x[n] = a^n u[n]$$

with $z-$transform:

$$X(z) = \frac{1}{1 - az^{-1}}, \quad |z| > |a|, |a| < 1$$

The analytical solution is obtained from the formula:

$$\hat{x}_[n] = \frac{a^n}{n}, \quad n > 0$$

The recursion solution is obtained from the recursion formula for a minimum-phase signal, $x_{\text{mnp}}[n]$, as:

$$\hat{x}_{\text{mnp}}[n] = \begin{cases} 0 & n < 0 \\ \log(x_{\text{mnp}}[0]) & n = 0 \\ \dfrac{x_{\text{mnp}}[n]}{x_{\text{mnp}}[0]} - \displaystyle\sum_{k=0}^{n-1} \left(\frac{k}{n}\right) \hat{x}_{\text{mnp}}[k] \frac{x_{\text{mnp}}[n-k]}{x_{\text{mnp}}[0]} & n > 0 \end{cases} \tag{1}$$

and finally the conventional solution (implemented using an FFT, a complex log, and an inverse FFT) on a finite duration version of $x[n]$, i.e., truncating $x[n]$ at some finite number of samples (one potential cause of errors), computing the FFT of the truncated sequence, computing the complex logarithm, and finally taking the IFFT of the resulting sequence.

Single Pole Cepstrum – GUI Design

The GUI for this exercise consists of two panels, 3 graphics panels, 1 title box and 3 buttons. The functionality of the two panels is:

1. one panel for the graphics display,
2. one panel for the single pole location, and for running the program.

The set of three graphics panels is used to display the following:

1. the impulse response of the single pole system,
2. the complex cepstrum of the single pole system computed using three analysis methods,

3. the error (difference) between the analytical method of cepstral analysis and the other two cepstral analysis methods.

The title box displays the information about the single pole parameter value. The functionality of the 3 buttons is:

1. an editable button that specifies the value of the single pole,
2. a pushbutton to run the code and display the results of the three methods of cepstral analysis on the three graphics panel displays,
3. a pushbutton to close the GUI.

Single Pole Cepstrum – Scripted Run

A scripted run of the program 'single_pole_cepstrum_GUI25.m' is as follows:

1. run the program 'single_pole_cepstrum_GUI25.m' from the directory 'matlab_gui\single_pole_cepstrum',
2. using the editable button, choose an initial value for the single pole location, a; default value of $a = 0.9$ used,
3. hit the 'Run Single Pole Cepstrum' button to compute and display the cepstrum of the single pole system as computed using the analytical formula, the recursion relation, and the DFT method, and plot the results on the three graphics panels,
4. experiment with different choices of real pole location,
5. hit the 'Close GUI' button to terminate the run.

An example of the graphical output obtained from this exercise is shown in Figure 1. The graphics panels show the impulse response of the single pole system (upper graphics panel), the complex cepstrum of the single pole system computed using the three cepstral analysis methods for a minimum phase system (middle graphics panel), and the differences between the analytically computed complex cepstrum and the complex cepstrum computed using the recursion formula and the DFT method (bottom graphics panel). Note that the scale on the bottom graphics panel is order of 10^{-17}, i.e., the errors are 17 orders of magnitude below the cepstral values.

Single Pole Cepstrum – Issues for Experimentation

1. run the scripted exercise above, and answer the following:
   - the middle graphics panels shows the complex cepstrum of a single pole system, computed using three different methods, namely the analytical method based on the root location, the recursion solution for a minimum phase signal, and the direct computation method using FFT and IFFT routines; why can't you see the three curves in the middle graphics panel?
   - the bottom graphics panel shows the differences between the complex cepstrum computed using the recursion method and the data sequence method, with the complex cepstrum computed using the analytical formula; what is the order of the largest differences for this one-pole system?
   - why are the differences the size that you obtain for this exercise?
   - vary the value of a from 0.1 to 0.19 in steps of 0.1. What happens to the error sequences as a varies over this range?
   - vary the value of a from 0.9 to 0.98 in steps of 0.02. What happens to the error sequence as a varies over this range? Can you explain this behavior?

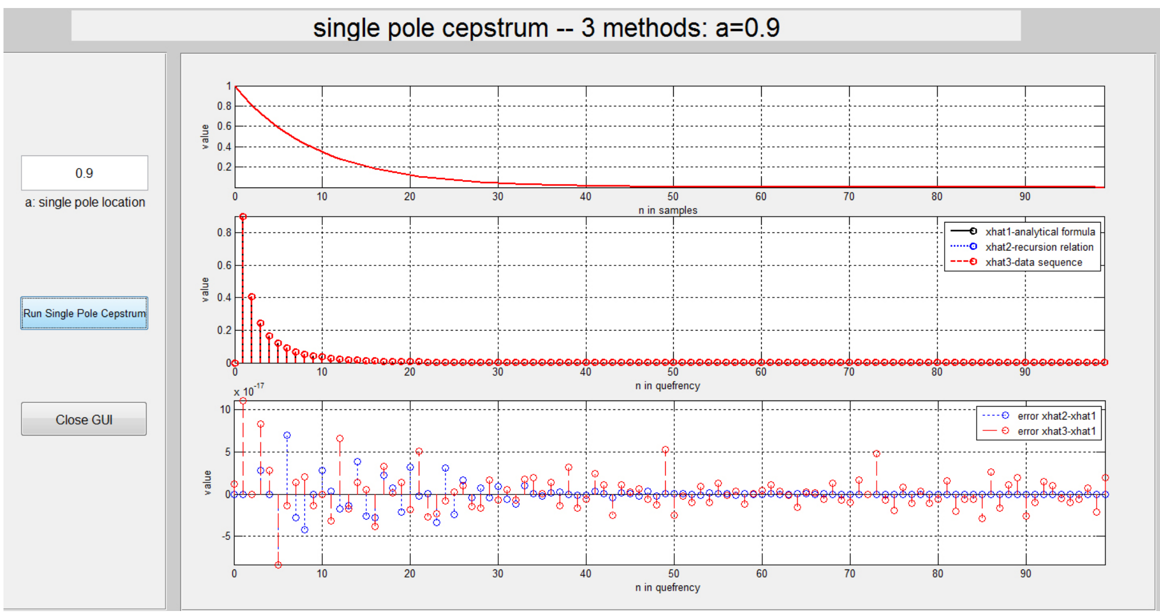

Figure 1: Cepstral computations for single pole model. The upper graphics panel shows the single pole system impulse response. The middle graphics panel shows the complex cepstrum computed using three different methods. The bottom graphics panels shows the differences between cepstral analyses.