Tomas Bata University in Zlín
Faculty of Applied Informatics

SELF-TUNING CONTROLLERS SIMULINK LIBRARY

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1 Introduction

Self-tuning controllers Simulink library contains various discrete single input single output (SISO) controllers that can be used to control systems of second and third order. All controllers contain on-line identification of controlled process and thus can be used also for time varying processes.

1.1 System requirements

The library has been designed under Matlab Release 12.1 (version 6.1) and Simulink version 4.1. For minimum and recommended system configuration see installation guide for Matlab. It is also possible to port the library to be used in previous versions of Simulink and Matlab, but in this case some functional limitations can occur. Dialog boxes used in library requires screen resolution at least 800x600 pixels.

1.2 Installation

Self-tuning controllers Simulink library is available in two versions:

- **stcsl_std.zip** – the standard version using the Matlab programming language (m-files)
- **stcsl_rtw.zip** – the version to be used with Real Time Workshop, using C language functions

To install the library simply unzip the appropriate file to an empty folder (for example using Winzip or pkunzip) and than add this folder to the matlab path. There are several possible approaches of adding a folder to the Matlab path (for example you can use **path** command), but the simplest way is to use the **Set Path** dialog, which can be accessed through the File menu.

1.3 Contact

Prof. Ing. Vladimir Bobal CSc. (bobal@ft.utb.cz)
Ing. Petr Chalupa (chalupa@ft.utb.cz)
Department of Control Theory
Institute of Information Technologies
Tomas Bata University in Zlin
Mostni 5139
760 01 Zlin
Czech Republic

2 How to use the library

The library file **stcsl_std.mdl** (or **stcsl_rtw.mdl**) contains 32 controller blocks, 20 of them can be used to control second order systems and 12 to control third order systems. Each controller block has 3 inputs and 2 outputs. The inputs are:

- Current process input (action value) *u_in*,
- Current process output *y*
- Reference value *w*

The controller outputs are:

- Current action value *u*
- Vector of current parameter estimations of controlled process

Input *u_in* should be as close as possible to actual control value (input to control process) to get the most appropriate results of the process identification. In simplest case this input is directly connected to controller output *u*. This situation is shows in following model:

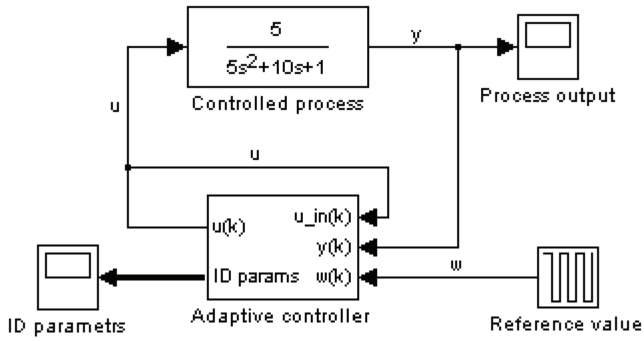


figure 1

Usually there are some limitations of action value in industrial applications and thus computed action value is not the same as input to the process. This situation is shown in sample model **circuit.mdl** in figure 2.

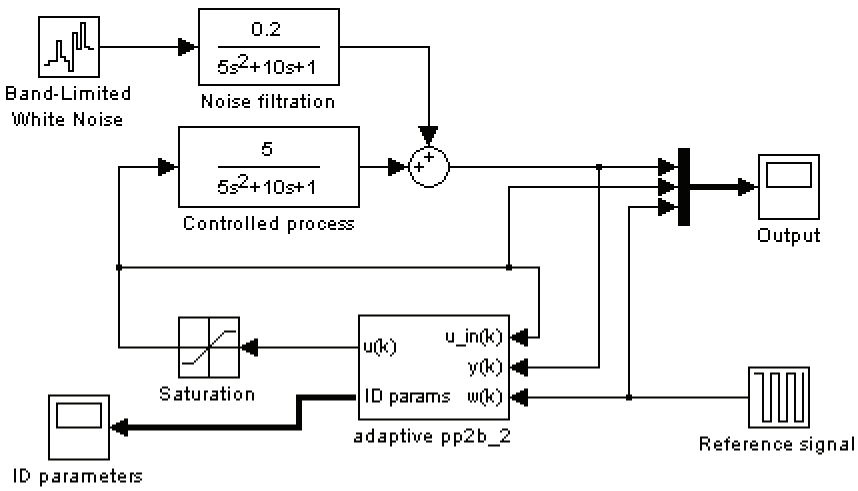


figure 2

To use blocks in your own Simulink models just copy the block from library into your model (for example using drag and drop operation), connect appropriate input and output signals, set block parameters (for example by double-clicking on block) and than you can start simulation. It is also possible to start with sample model **circuit.mdl** and modify it to represent process you want to simulate. Controller parameters can be divided into two groups: the parameters common to all blocks and the controller-specific parameters. These two groups will be discussed in following two chapters.

3 Common controllers parts and parameters

This section describes the controllers' parts and their parameters that are common for several or all controllers in the library. There are three subsections: description of on-line identification methods used in all controllers in the library, computation of ultimate gain and period used in controller based on Ziegler-Nichols method and the list of common controller parameters.

3.1 On-line identification methods

This section describes methods used in the library for an on-line process identification. You can select one of three offered methods: Least squares method (LSM), LSM with exponential forgetting and LSM with adaptive directional forgetting. All these methods can be used for the discrete on-line identification of processes that are described by the following transfer function:

$$G(z)=\frac{B\left(z^{-1}\right)}{A\left(z^{-1}\right)}=\frac{b_1 z^{-1}+b_2 z^{-2}+\ldots+b_m z^{-m}}{1+a_1 z^{-1}+a_2 z^{-2}+\ldots+a_n z^{-n}} z^{-d} \tag{1}$$

The estimated output of the process in step *k* ($\hat{y}_k$) is computed on base of the previous process inputs *u* and outputs *y* according to the equation (2):

$$\begin{aligned}\hat{y}_k &= -\hat{a}_1 y_{k-1}-\ldots-\hat{a}_n y_{k-n} \\ &\quad +\hat{b}_1 u_{k-d-1}+\ldots+\hat{b}_m u_{k-d-m}\end{aligned} \tag{2}$$

where $\hat{a}_1, \ldots, \hat{a}_n, \hat{b}_1, \ldots, \hat{b}_m$ are the current estimations of process parameters. This equation can be also written in vector form, which is more suitable for further work - see equation (3).

$$\begin{aligned}\hat{y}_k &= \boldsymbol{\Theta}_{k-1}^T \cdot \boldsymbol{\Phi}_k \\ \boldsymbol{\Theta}_{k-1} &= \left[\hat{a}_1,\ldots,\hat{a}_n,\hat{b}_1,\ldots,\hat{b}_m\right]^T \\ \boldsymbol{\Phi}_k &= \left[-y_{k-1},\ldots,-y_{k-n},u_{k-d-1},\ldots,u_{k-d-m}\right]^T\end{aligned} \tag{3}$$

The vector $\boldsymbol{\Theta}_{k-1}$ contains the process parameter estimations computed in previous step and the vector $\boldsymbol{\Phi}_k$ contains output and input values for computation of current output y_k. This vectors record is used in description of individual identification methods in further sections.

Recursive least square method (LSM)

Least square methods are based on minimisation of the sum of prediction errors squares:

$$J_k=\sum_{i=1}^{k}\left(y_i-\boldsymbol{\Theta}_k^T \boldsymbol{\Phi}_i\right)^2 \tag{4}$$

Where y_I is process output in i-th step and the product $\Theta_k^T \Phi_i$ represents predicted process output. Solving this equation leads to the recursive version of least square method where vector of parameters estimations is updated in each step according to equation (5):

$$\boldsymbol{\Theta}_k=\boldsymbol{\Theta}_{k-1}+\frac{\boldsymbol{C}_{k-1} \cdot \boldsymbol{\Phi}_k}{1+\boldsymbol{\Phi}_k^T \cdot \boldsymbol{C}_{k-1} \cdot \boldsymbol{\Phi}_k} \cdot\left(y_k-\boldsymbol{\Theta}_{k-1}^T \boldsymbol{\Phi}_k\right) \tag{5}$$

The covariance matrix C is then updated in each step as defined by the equation (6):

$$\boldsymbol{C}_k=\boldsymbol{C}_{k-1}-\frac{\boldsymbol{C}_{k-1} \cdot \boldsymbol{\Phi}_k \cdot \boldsymbol{\Phi}_k^T \cdot \boldsymbol{C}_{k-1}}{1+\boldsymbol{\Phi}_k^T \cdot \boldsymbol{C}_{k-1} \cdot \boldsymbol{\Phi}_k} \tag{6}$$

Initial value of matrix **C** determines influence of initial parameter estimations to the identification process.

LSM with exponential forgetting

The main disadvantage of this pure recursive least square method is an absence of signal weighting. Each input and output affect result by the same weight, but actual process parameters can change in time. Thus newer inputs and outputs should affect output more than older values. This problem can be solved by exponential forgetting method, which uses forgetting coefficient φ and decreases the weights of the data in previous steps. Weights φ^{k-i} are assigned to values u_i and y_i. Parameter estimations are computed according to following equations:

$$\boldsymbol{\Theta}_k = \boldsymbol{\Theta}_{k-1} + \frac{\boldsymbol{C}_{k-1} \cdot \boldsymbol{\Phi}_k}{\varphi + \boldsymbol{\Phi}_k^T \cdot \boldsymbol{C}_{k-1} \cdot \boldsymbol{\Phi}_k} \cdot \left(y_k - \boldsymbol{\Theta}_{k-1}^T \boldsymbol{\Phi}_k \right) \tag{7}$$

$$\boldsymbol{C}_k = \frac{1}{\varphi} \left(\boldsymbol{C}_{k-1} - \frac{\boldsymbol{C}_{k-1} \cdot \boldsymbol{\Phi}_k \cdot \boldsymbol{\Phi}_k^T \cdot \boldsymbol{C}_{k-1}}{\varphi + \boldsymbol{\Phi}_k^T \cdot \boldsymbol{C}_{k-1} \cdot \boldsymbol{\Phi}_k} \right) \tag{8}$$

LSM with adaptive directional forgetting

The exponential forgetting method can be further improved by adaptive directional forgetting which changes forgetting coefficient with respect to changes of input and output signal. Process parameters are updated using recursive equation:

$$\boldsymbol{\Theta}_k = \boldsymbol{\Theta}_{k-1} + \frac{\boldsymbol{C}_{k-1} \cdot \boldsymbol{\Phi}_k}{1 + \xi} \cdot \left(y_k - \boldsymbol{\Theta}_{k-1}^T \boldsymbol{\Phi}_k \right) \tag{9}$$

where

$$\xi = \boldsymbol{\Phi}_k^T \cdot \boldsymbol{C}_{k-1} \cdot \boldsymbol{\Phi}_k \tag{10}$$

Matrix C is updated in each step according to equation (11):

$$\boldsymbol{C}_k = \boldsymbol{C}_{k-1} - \frac{\boldsymbol{C}_{k-1} \cdot \boldsymbol{\Phi}_k \cdot \boldsymbol{\Phi}_k^T \cdot \boldsymbol{C}_{k-1}}{\varepsilon^{-1} + \xi} \tag{11}$$

where

$$\varepsilon = \varphi_{k-1} - \frac{1 - \varphi_{k-1}}{\xi} \tag{12}$$

Forgetting coefficient is updated as follows:

$$\varphi_k = \frac{1}{1 + (1 + \rho)\left\{ \ln(1 + \xi) + \left[\frac{(\upsilon_k + 1)\eta}{1 + \xi + \eta} - 1 \right] \frac{\xi}{1 + \xi} \right\}} \tag{13}$$

where

$$\upsilon_k = \varphi_{k-1}(\upsilon_{k-1} + 1) \tag{14}$$

$$\eta = \frac{\left(y_k - \boldsymbol{\Theta}_{k-1}^T \boldsymbol{\Phi}_k \right)^2}{\lambda_k} \tag{15}$$

$$\lambda_k = \varphi_{k-1} \left[\lambda_{k-1} + \frac{\left(y_k - \boldsymbol{\Theta}_{k-1}^T \boldsymbol{\Phi}_k \right)^2}{1 + \xi} \right] \tag{16}$$

All described on-line identification methods are realized in s-function **sid** where further details can be found. See help text for this function (type “**help sid**” in Matlab command prompt) or study the code of **sid.m** or **sid.c**.

3.2 Computation of ultimate gain and period

Self-tuning controllers based on Ziegler-Nichols method needs to know process ultimate gain and period in each step and these parameters must be calculated in each step. The basic idea of this calculation is to find feedback gain to reach stability border of closed loop. For process transfer function defined by equation (1), the characteristic equation of closed loop is:

$$A(z) + K_p \cdot B(z) = 0 \qquad (17)$$

where K_p is feedback gain.

When the closed system is on the stability border, one or more roots of its characteristic equation are on stability border and the other roots are stable. In complex variable *z*, the stable region is the inner of unit circle, the stability border is unit circle and rest is unstable region. The ultimate root *z=1* must be omitted in calculation because this is asymptotic stability border and thus ultimate period cannot be calculated.

The are two possible types of roots positions when calculating ultimate gain and ultimate period:

- Two complex conjugated roots on unit circle and other roots are not unstable
- Root *z = -1* and other roots are not unstable

The algorithm of computation ultimate gain and ultimate period is placed in file **ultim.m** or **ultim.c.** For further details see help of this file (type "**help ultim**" in Matlab command prompt) or study the code.

3.3 List of common controllers parameters

Following controller parameters are common to all self-tuning controllers in the library. The common parameters are: Sample time T0, Identification type, ID Initial parameters estimations, ID Initial covariance matrix C, ID Initial phi (forgetting coefficient), ID Initial lambda, ID Initial rho and ID Initial nu The table bellow contains a short description of each of these parameters.

<table>
<tr><th>Group</th><th>Parameter</th><th>Description</th></tr>
<tr><td></td><td>Sample time T0</td><td>Defines sample time of self-tuning controller. This value is used for both parts of controller: process identification and calculation of action value</td></tr>
<tr><td rowspan="7">On-line Recursive Identification</td><td>Identification type</td><td>You can select from following on-line identification methods:
• Least squares method (LSM) – The simplest method where all process input-output pairs affect identified parameters with the same weight.
• LSM with exponential forgetting – The newest process input-output pairs affect identified parameters more than older pairs. This method can be used for systems with time varying parameters.
• LSM with adaptive directional forgetting – The most sophisticated method, the weight of current process input-output pair is determined with respect to changes of input and output signal. This method is useful for systems with time varying parameters.</td></tr>
<tr><td>ID Initial parameters estimations</td><td>Initial process parameter estimation used by on-line identification. This is a column vector of parameters in the form [a1; a2; b1; b2] for most second order controllers, [a1; a2; b1] for Dahlin’s controller and [a1; a2; a3; b1; b2; b3] for third order systems. Details are given in description of individual controller in next chapter.</td></tr>
<tr><td>ID Initial covariance matrix C</td><td>Initial value of the covariance matrix used in on-line identification process. This must be a square positive definite matrix with dimension the same as number of identified parameters.
Usually a diagonal matrix is used in the form G*I, where I is identity matrix and G is gain. The gain than determines the influence of initial parameter estimations to the identification process: the greater gain, the smaller influence of initial estimations.</td></tr>
<tr><td>ID Initial phi (forgetting coefficient)</td><td>Initial value of forgetting coefficient φ used in on-line identification of controlled process. This parameter is used only if Identification type is LSM with exponential forgetting or LSM with adaptive directional forgetting and should be in range $0 < \varphi \leq 1$
When LSM with exponential forgetting is used, this parameter determines the forgetting rate of older process input-output pairs: the smaller φ, the smaller influence of older pairs to the current parameter estimations. When LSM with adaptive directional forgetting is used, then φ is changing during identification process according to process of input (u) and output (y) values</td></tr>
<tr><td>ID Initial lambda</td><td>Initial value of parameter λ used in on-line identification of controlled process. This parameter is used only if Identification type is LSM with adaptive directional forgetting.</td></tr>
<tr><td>ID Initial rho</td><td>Initial value of parameter ρ used in on-line identification of controlled process. This parameter is used only if Identification type is LSM with adaptive directional forgetting.</td></tr>
<tr><td>ID Initial nu</td><td>Initial value of parameter υ used in on-line identification of controlled process. This parameter is used only if Identification type is LSM with adaptive directional forgetting.</td></tr>
</table>

4 Controllers reference

This chapter describes individual self-tuning controllers included in the library and their parameters. The library contains 29 self-tuning controllers: 18 of them are designed to control second order processes and remaining 11 are designed to control third order processes.

Each self-tuning controller can be divided into three parts:

- On-line identification – Recursive identification using least squares method on base of parameters described in previous chapter. This block is realized by Simulink s-function **sid** and to get more detailed help type **help sid** in matlab command prompt.
- Calculation of controller parameters – The main function of each controller used to compute coefficients of control law on base of identified process parameters and controller-specific parameters. This function is specific to each controller. The function name is the same as controller name thus to get detailed help of each function use Matlab help command (e.g. **help ba2**).
- Calculation of output (action value) – This block computes controller output (action value) on base of reference value, process output and coefficients computed by previous block. There are 4 s-functions which realizes the computation for different controllers:
  - **scqp** used for feed-back controllers ba2, da2, pp2a_1, pp2a_2, zn2br, zn2fd, zn2fr, zn2pi, zn2tr, zn3br, zn3fd, zn3fr, zn3pi and zn3tr;
  - **scrqp** used for feed-back feed-forward controllers db2s, db2w, mv2, pp2b_1, pp2b_2, zn2pd, zn2tak, db3s, db3w, zn3pd and zn3tak;
  - **scast** used for Astrom's controllers zn2ast and zn3ast
  - **scfpd** used for four point difference controllers zn2fpd and zn3fpd.
  - **scfbfw** used for 2DOF controllers pp2chp, pp3chp and pp2lq

  To get detailed help of these s-functions use help command in Matlab command prompt (e.g. **help scqp**)

The structure of all controller names is as follows: **xxnyyyy,** where **xx** - controller type, **n** = 2 or 3 process model order and **yyyy** - controller further details. The library consists of following self-tuning controllers (their parameters are described in following sections):

Contr. No.	Controller algorithm	Input parameters Controller type
1 zn2fr zn3fr	$u(k)=q_0e(k)+q_1e(k-1)+q_2e(k-2)+u(k-1)$ $q_0=K_P\left(1+\frac{T_D}{T_0}\right);\quad q_1=-K_P\left(1-\frac{T_0}{T_I}+2\frac{T_D}{T_0}\right);\quad q_2=K_P\frac{T_D}{T_0}$	K_{Pu}; T_u; T_0 PID controller using forward rectangular method
2 zn2br zn3br	$u(k)=q_0e(k)+q_1e(k-1)+q_2e(k-2)+u(k-1)$ $q_0=K_P\left(1+\frac{T_0}{T_I}+\frac{T_D}{T_0}\right);\quad q_1=-K_P\left(1+2\frac{T_D}{T_0}\right);\quad q_2=K_P\frac{T_D}{T_0}$	K_{Pu}; T_u; T_0 PID controller using backward rectangular method
3 zn2tr zn3tr	$u(k)=q_0e(k)+q_1e(k-1)+q_2e(k-2)+u(k-1)$ $q_0=K_P\left(1+\frac{T_0}{2T_I}+\frac{T_D}{T_0}\right);\quad q_1=-K_P\left(1-\frac{T_0}{2T_I}+\frac{2T_D}{T_0}\right);\quad q_2=K_P\frac{T_D}{T_0}$	K_{Pu}; T_u; T_0 PID controller using trapezoidal rectangular method

4 zn2pd zn3pd	$$u(k)=K_P\left\{w(k)-y(k)+\frac{T_D}{T_0}\left[y(k-1)-y(k)\right]\right\}$$ $K_P=0.4K_{Pu}$; $T_D=\frac{T_U}{20}$	K_{Pu}; T_u; T_0 PD controller
5 zn2pi zn3pi	$$u(k)=q_0e(k)+q_1e(k-1)+u(k-1)$$ $q_0=K_P\left(1+\frac{T_0}{2T_I}\right)$; $q_1=-K_P\left(1-\frac{T_0}{2T_I}\right)$	K_{Pu}; T_u; T_0 PI controller
6 zn2fd zn3fd	$$u(k)=q_0e(k)+q_1e(k-1)+q_2e(k-2)-p_1u(k-1)-p_2u(k-2)$$ $p_1=\dfrac{-4\frac{T_f}{T_0}}{\frac{2T_f}{T_0}+1}$; $p_2=\dfrac{\frac{2T_f}{T_0}-1}{\frac{2T_f}{T_0}+1}$; $q_0=\dfrac{K_P+2K_P\frac{T_f+T_D}{T_0}+\frac{K_PT_0}{2T_I}(\frac{2T_f}{T_0}+1)}{\frac{2T_f}{T_0}+1}$ $q_1=\dfrac{\frac{K_PT_0}{2T_I}-4K_P\frac{T_f+T_D}{T_0}}{\frac{2T_f}{T_0}+1}$; $q_2=\dfrac{\frac{T_f}{T_0}(2K_P-\frac{K_PT_0}{T_I})+2\frac{K_PT_D}{T_0}+\frac{K_PT_0}{2T_I}-K_P}{\frac{2T_f}{T_0}+1}$; $T_f=\frac{T_D}{\alpha}$; $\alpha\in\langle 3;20\rangle$	K_{Pu}; T_u; T_0; T_f PID controller using filtration of D-component, Tustin approximation
7 zn2tak zn3tak	$$u(k)=K_R[y(k-1)-y(k)]+K_I[w(k)-y(k)]+K_D[2y(k-1)-y(k-2)-y(k)]+ +u(k-1)$$ $K_R=0.6K_{Pu}-\frac{K_I}{2}$; $K_I=\frac{1.2K_{Pu}T_0}{T_u}$; $K_D=\frac{3K_{Pu}T_u}{40T_0}$	K_{Pu}; T_u; T_0 PID - Takahashi's controller
8 zn2ast zn3ast	$$u(k)=u_{PI}(k)+u_D(k)$$ $$u_{PI}(k)=K_P\left[y(k-1)-y(k)\right]+ +\frac{K_PT_0}{2T_I}\left[e(k)+e(k-1)\right]+\beta K_P\left[w(k)-w(k-1)\right]+u_{PI}(k-1)$$ $$u_D(k)=K_P\frac{T_D\alpha}{T_D+T_0\alpha}\left[y(k-1)-y(k)\right]+\frac{T_D}{T_D+T_0\alpha}u_D(k-1)$$	K_{Pu}; T_u; T_0 PID - Åström's controller
9 zn2fpd zn3fpd	$$u(k)=K_P\left\{e(k)-e(k-1)+\frac{T_0}{T_I}e(k)+ +\frac{T_D}{6T_0}\left[e(k)+2e(k-1)-6e(k-2)+2e(k-3)+e(k-4)\right]\right\}+u(k-1)$$	K_{Pu}; T_u; T_0 PID controller using backward rectangular method, replacing derivation by a four-point difference
10 ba2	$$u(k)=q_0e(k)+q_1e(k-1)+q_2e(k-2)+u(k-1)$$ $q_0=\frac{k_I}{b_0}$; $q_1=q_0a_1=\frac{k_I}{b_0}a_1$; $q_2=q_0a_2=\frac{k_I}{b_0}a_2$ $k_I=\frac{1}{2d-1}$ ($\gamma=0$); $k_I=\frac{1}{2d(1+\gamma)(1-\gamma)}$ ($\gamma>0$); $\gamma=\frac{b_1}{b_0}$	*d* - time delay PID - Bányász - Keviczky's controller

11 da2	$u(k)=K_P\left\{e(k)-e(k-1)+\frac{T_0}{T_I}e(k)+\frac{T_D}{T_0}\left[e(k)-2e(k-1)+e(k-2)\right]\right\}+u(k-1)$ $K_P=-\frac{(a_1+2a_2)Q}{b_1}$; $T_I=-\frac{T_0}{\frac{1}{a_1+2a_2}+1+\frac{T_D}{T_0}}$ $T_D=\frac{T_0 a_2 Q}{K_P b_1}$; $Q=1-e^{-\frac{T_0}{B}}$	B - adjustment factor T_0 PID - Dahlin's controller
12 pp2a_1	$u(k)=q_0e(k)+q_1e(k-1)+q_2e(k-2)+(1-\gamma)u(k-1)+\gamma u(k-2)$ $q_0=\frac{1}{b_1}(d_1+1-a_1-\gamma)$; $q_1=\frac{a_2}{b_2}-q_2(\frac{b_1}{b_2}-\frac{a_1}{a_2}+1)$; $q_2=-\frac{s_1}{r_1}$; $\gamma=q_2\frac{b_2}{a_2}$ $s_1=a_2\left[(b_1+b_2)(a_1b_2-a_2b_1)+b_2(b_1d_2-b_2d_1-b_2)\right]$; $r_1=(b_1+b_2)(a_1b_1b_2+a_2b_1^2+b_2^2)$	ω_n - natural frequency ξ - damping factor PID A-1 pole placement controller
13 pp2a_2	$u(k)=q_0e(k)+q_1e(k-1)+q_2e(k-2)+(1-\gamma)u(k-1)+\gamma u(k-2)$ $q_0=\frac{r_2-r_3}{r_1}$; $q_1=-\frac{r_4+r_5}{r_1}$; $q_2=\frac{x_4+\gamma a_2}{b2}$; $\gamma=\frac{r_6}{r_1}$ $r_1=(b_1+b_2)(a_1b_1b_2+a_2b_1^2+b_2^2)$; $r_2=x_1(b_1+b_2)(a_1b_2-a_2b_1)$ $r_3=b_1^2x_4-b_2\left[b_1x_3-b_2(x_1+x_2)\right]$; $r_4=a_1\left[b_1^2x_4+b_2^2x_1-b_1b_2(x_2+x_3)\right]$ $r_5=(b_1+b_2)\left[a_2(b_1x_2-b_2x_1)-b_1x_4+b_2x_3\right]$; $r_6=b_1(b_1^2x_4-b_1b_2x_3+b_2^2x_2)-b_2^3x_1$	α, ω - real and imaginary component of the pole PID A-2 pole placement controller
14 pp2b_1	$u(k)=-\left[(q_0'+\beta)y(k)-(q_0'+q_2')y(k-1)+q_2'y(k-2)\right]-$ $-(\gamma-1)u(k-1)+\gamma u(k-2)+\beta w(k)$ $q_0'=q_2'(\frac{b_1}{b_2}-\frac{a_1}{a_2})-\frac{a_2}{b_2}$; $q_2'=\frac{s_1}{r_1}$; $\gamma=q_2'\frac{b_2}{a_2}$; $\beta=\frac{1}{b_1}(d_1+1-a_1-\gamma-b_1q_0')$ $s_1=a_2\left\{b_2\left[a_1(b_1+b_2)+b_1(d_2-a_2)-b_2(d_1+1)\right]-a_2b_1^2\right\}$ $r_1=(b_1+b_2)(a_1b_1b_2-a_2b_1^2-b_2^2)$	ω_a - natural frequency ξ - damping factor PID B-1 pole placement controller
15 pp2b_2	$u(k)=-\left[(q_0'+\beta)y(k)-(q_0'+q_2')y(k-1)+q_2'y(k-2)\right]-$ $-(\gamma-1)u(k-1)+\gamma u(k-2)+\beta w(k)$ $q_0'=-\frac{r_2-r_3+r_4}{r_1}$; $q_2'=\frac{r_6+r_7}{r_1}$; $\gamma=\frac{r_5}{r_1}$; $\beta=\frac{x_1+x_2-x_3+x_4}{b_1+b_2}$ $r_1=(b_1+b_2)(a_1b_1b_2-a_2b_1^2-b_2^2)$; $r_2=a_1b_2\left[b_1(x_2+x_3+x_4)-b_2x_1\right]$ $r_3=a_2b_1\left[b_2x_1-b_1(x_2-x_3+x_4)\right]$; $r_4=(b_1+b_2)\left[b_1x_4+b_2(-x_3-x_4)\right]$ $r_5=b_1(b_1^2x_4-b_1b_2x_3+b_2^2x_2)-b_2^3x_1$; $r_6=b_1^2(-a_2x_3+a_1x_4-a_2x_4)$ $r_7=b_2\left[b_1(a_1x_4+a_2x_2-x_4)-b_2(a_2x_1+x_4)\right]$	α, ω - real and imaginary component of the pole PID B-2 pole placement controller
16 db2w db3w	$u(k)=r_0w(k)-q_0y(k)-q_1y(k-1)-q_2y(k-2)-$ $-p_1u(k-1)-p_2u(k-2)$	Dead-beat controller (weak version)
17 db2s db3s	$u(k)=r_0w(k)-q_0y(k)-q_1y(k-1)-q_2y(k-2)-$ $-p_1u(k-1)-p_2u(k-2)$	Dead-beat controller (strong version)

18 mv2	$u(k)=\frac{1}{q}\left[a_1y(k-1)+a_2y(k-2)-b_1u(k-1)-b_2u(k-2)+w(k)\right]+u(k-1)$	*q* - penalisation factor Minimum variance controller
19 pp2chp pp3chp	$u(k)=r_0w(k)+r_1w(k-1)+r_2w(k-2)-$ $-q_0y(k)-q_1y(k-1)-q_2y(k-2)-p_1u(k-1)-p_2u(k-2)$ $A(z^{-1})P(z^{-1})+B(z^{-1})Q(z^{-1})=D(z^{-1})$ $B(z^{-1})R(z^{-1})+F(z^{-1})S(z^{-1})=D(z^{-1})$	*D* – characteristic polynomial, Reference signal type T_0 Generic pole placement controller
20 pp2lq	$u(k)=r_0w(k)+r_1w(k-1)+r_2w(k-2)-$ $-q_0y(k)-q_1y(k-1)-q_2y(k-2)-p_1u(k-1)-p_2u(k-2)$ $A(z^{-1})P(z^{-1})+B(z^{-1})Q(z^{-1})=D(z^{-1})$ $B(z^{-1})R(z^{-1})+F(z^{-1})S(z^{-1})=D(z^{-1})$ $D(z^{-1})=1+d_1z^{-1}+d_2z^{-2}$ $d_1=\frac{m_1}{\delta+m_2}$ $\quad d_2=\frac{m_2}{\delta}$ $\delta=\frac{\lambda+\sqrt{\lambda^2-4m_2^{\,2}}}{2}$ $\quad \lambda=\frac{m_0}{2}-m_2+\sqrt{\left(\frac{m_0}{2}+m_2\right)^2-m_1^{\,2}}$ $m_0=\varphi(1+a_1^{\,2}+a_2^{\,2})+b_1^{\,2}+b_2^{\,2}$ $\quad m_1=\varphi(a_1+a_1a_2)+b_1b_2$ $\quad m_2=\varphi a_2$	*Φ* – weight of controller output signal in LQ criterion Reference signal type T_0 Controller based on minimisation of LQ criterion
21 pp2c2dof pp3c2dof	$u(k)=r_0w(k)-q_0y(k)-q_1y(k-1)-q_2y(k-2)-q_3y(k-3)+$ $+(1-p_1)u(k-1)+(p_1-p_2)u(k-2)+p_2u(k-3)$ $A(z^{-1})P(z^{-1})K(z^{-1})+B(z^{-1})Q(z^{-1})=D(z^{-1})$ $D(z^{-1})=1+d_1z^{-1}+d_2z^{-2}$ $r_0=\frac{1+d_1+d_2+\ldots+d_{2n}}{b_1+b_2+\ldots+b_n}$	ω_n - natural frequency ξ - damping factor 2DOF pole placement controller with compensator

Notes:

m-file name: **xx(x)a(yyyy).m**: **xx(x)** - controller type, **n** = 2 or 3 - process model order, **(yyyy)** - controller further details

Controllers number 1, 2, 3, 4, 5, 6, 7, 8 and 9:

$$K_P=0.6K_{Pu};\; T_I=0.5T_u;\qquad T_D=0.125T_u$$

Controllers number 12, 14 and 21:

$$d_1=-2\exp(-\xi\omega_nT_0)\cos(\omega_nT_0\sqrt{1-\xi^2})\text{ for } \xi \le 1,$$

$$d_1=-2\exp(-\xi\omega_nT_0)\cosh(\omega_nT_0\sqrt{1-\xi^2})\text{ for } \xi > 1,$$

$$d_2=\exp(-2\xi\omega_nT_0)$$

Controllers number 13 and 15:

$$x_1=c+1-a_1;\qquad x_2=d+a_1-a_2;\qquad x_3=f+a_2;\qquad x_4=g$$

$c = -4\alpha;$ $d = 6\alpha^2 + \omega^2;$ $f = -2\alpha(2\alpha^2 + \omega^2);$ $g = \alpha^2(\alpha^2 + \omega^2)$

4.1 zn2fr

Ziegler-Nichols PID controller for processes of second order. Controller is based on forward rectangular method of discretization.

Transfer function of the controlled system:

$$G(z)=\frac{b_1 z^{-1}+b_2 z^{-2}}{1+a_1 z^{-1}+a_2 z^{-2}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{b}_1,\hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:

None.

Control law:

$$u_k=K_P\left[e_k-e_{k-1}+\frac{T_0}{T_I}e_{k-1}+\frac{T_D}{T_0}\left(e_k-2e_{k-1}+e_{k-2}\right)\right]+u_{k-1}$$

where e_k is control error ($e_k=w_k-y_k$). This form of control law can be transformed to feedback form:

$$u_k=q_0e_k+q_1e_{k-1}+q_2e_{k-2}+u_{k-1}$$

Controller parameters are calculated using following equations:

$$q_0=K_P\left(1+\frac{T_D}{T_0}\right)\qquad q_1=-K_P\left(1-\frac{T_0}{T_I}+2\frac{T_D}{T_0}\right)\qquad q_2=K_P\frac{T_D}{T_0}$$

$$K_P=0.6K_{Pu}\qquad T_I=0.5T_u\qquad T_D=0.125T_u$$

Variables K_{Pu} and T_u are ultimate gain and ultimate period respectively.

Source code:

sid on-line identification s-function

zn2fr function for computation of controller parameters

scqp controller s-function

See also:

On-line identification methods
Computation of ultimate gain and period
List of common controllers parameters

4.2 zn2br

Ziegler-Nichols PID controller for processes of second order. The controller is based on backward rectangular method of discretization.

Transfer function of the controlled system:

$$G(z)=\frac{b_1 z^{-1}+b_2 z^{-2}}{1+a_1 z^{-1}+a_2 z^{-2}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:

None.

Control law:

$$u_k = K_P\left[e_k - e_{k-1} + \frac{T_0}{T_I}e_k + \frac{T_D}{T_0}\left(e_k - 2e_{k-1} + e_{k-2}\right)\right] + u_{k-1}$$

where e_k is control error ($e_k = w_k - y_k$). This form of control law can be transformed to feedback form:

$$u_k = q_0 e_k + q_1 e_{k-1} + q_2 e_{k-2} + u_{k-1}$$

Controller parameters are calculated using following equations:

$$q_0 = K_P\left(1+\frac{T_0}{T_I}+\frac{T_D}{T_0}\right) \qquad q_1 = -K_P\left(1+2\frac{T_D}{T_0}\right) \qquad q_2 = K_P\frac{T_D}{T_0}$$

$$K_P = 0.6K_{Pu} \qquad T_I = 0.5T_u \qquad T_D = 0.125T_u$$

Variables K_{Pu} and T_u are ultimate gain and ultimate period respectively.

Source code:

sid on-line identification s-function

zn2br function for computation of controller parameters

scqp controller s-function

See also:

On-line identification methods
Computation of ultimate gain and period
List of common controllers parameters

4.3 zn2tr

Ziegler-Nichols PID controller for processes of second order. Controller is based on trapezoidal method of discretization.

Transfer function of the controlled system:

$$G(z)=\frac{b_1 z^{-1}+b_2 z^{-2}}{1+a_1 z^{-1}+a_2 z^{-2}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{b}_1,\hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:

None.

Control law:

$$u_k=K_P\left[e_k-e_{k-1}+\frac{T_0}{T_I}\frac{e_k-e_{k-1}}{2}+\frac{T_D}{T_0}\left(e_k-2e_{k-1}+e_{k-2}\right)\right]+u_{k-1}$$

where e_k is control error ($e_k=w_k-y_k$). This form of control law can be transformed to feedback form:

$$u_k=q_0 e_k+q_1 e_{k-1}+q_2 e_{k-2}+u_{k-1}$$

Controller parameters are calculated using following equations:

$$q_0=K_P\left(1+\frac{T_0}{2T_I}+\frac{T_D}{T_0}\right)\qquad q_1=-K_P\left(1-\frac{T_0}{2T_I}+\frac{2T_D}{T_0}\right)\qquad q_2=K_P\frac{T_D}{T_0}$$

$$K_P=0.6K_{Pu}\qquad T_I=0.5T_u\qquad T_D=0.125T_u$$

Variables K_{Pu} and T_u are ultimate gain and ultimate period respectively.

Source code:

sid on-line identification s-function

zn2tr function for computation of controller parameters

scqp controller s-function

See also:

On-line identification methods
Computation of ultimate gain and period
List of common controllers parameters

4.4 zn2pd

Ziegler-Nichols PD controller for processes of second order. Controller uses reference variable (w) just in proportional component.

Transfer function of the controlled system:

$$G(z)=\frac{b_1z^{-1}+b_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{b}_1,\hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:
None.

Control law:

$$u_k=K_P\left[w_K-y_k+\frac{T_D}{T_0}\left(y_{k-1}-y_k\right)\right]$$

This form of control law can be transformed to feedback feedforward form:

$$u_k=r_0w_k-q_0y_k-q_1y_{k-1}$$

where controller parameters r_0, q_0 and q_1 are calculated using following equation:

$$r_0=K_P \qquad q_0=K_P\left(1+\frac{T_D}{T_0}\right) \qquad q_1=-K_P\frac{T_D}{T_0}$$

$$K_P=0.4K_{Pu} \qquad T_D=\frac{T_u}{20}$$

Variables K_{Pu} and T_u are ultimate gain and ultimate period respectively.

Source code:

sid on-line identification s-function

zn2pd function for computation of controller parameters

scrqp controller s-function

See also:
On-line identification methods
Computation of ultimate gain and period
List of common controllers parameters

4.5 zn2pi

Ziegler-Nichols PI controller for processes of second order. Controller is based on trapezoidal method of discretization.

Transfer function of the controlled system:

$$G(z)=\frac{b_1 z^{-1}+b_2 z^{-2}}{1+a_1 z^{-1}+a_2 z^{-2}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:

None.

Control law:

$$u_k = K_P\left[e_k - e_{k-1} + \frac{T_0}{T_I}\frac{e_k - e_{k-1}}{2}\right] + u_{k-1}$$

where e_k is control error ($e_k = w_k - y_k$). This form of control law can be transformed to feedback form:

$$u_k = q_0 e_k + q_1 e_{k-1} + u_{k-1}$$

Controller parameters are calculated using following equations:

$$q_0 = K_P\left(1+\frac{T_0}{2T_I}\right) \qquad q_1 = -K_P\left(1-\frac{T_0}{2T_I}\right)$$

$$K_P = 0.6K_{Pu} \qquad T_I = 0.5T_u$$

Variables K_{Pu} and T_u are ultimate gain and ultimate period respectively.

Source code:

sid on-line identification s-function

zn2pi function for computation of controller parameters

scqp controller s-function

See also:

[On-line identification methods]
[Computation of ultimate gain and period]
[List of common controllers parameters]

4.6 zn2fd

Ziegler-Nichols controller for second order processes with filtration of D-component using Tustin approximation.

Transfer function of the controlled system:

$$G(z)=\frac{b_1 z^{-1}+b_2 z^{-2}}{1+a_1 z^{-1}+a_2 z^{-2}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{b}_1,\hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:

Parameter	Description
alfa - filtration coefficient	Filtration coefficient α used by filter of process output signal. The time constant of the filter is: $T_d=\frac{T_D}{\alpha}$ where usually 3 < α < 20

Control law:

$$u_k=q_0e_k+q_1e_{k-1}+q_2e_{k-2}-p_1u_{k-1}-p_2u_{k-2}$$

where e_k is control error ($e_k=w_k-y_k$) and controller parameters are calculated using following equations:

$$q_0=K_P\frac{1+2\left(c_f+c_d\right)+\frac{c_i}{2}\left(1+2c_f\right)}{1+2c_f} \qquad q_1=K_P\frac{\frac{c_i}{2}-4\left(c_f+c_d\right)}{1+2c_f}$$

$$q_2=K_P\frac{c_f\left(2-c_i\right)+2c_d+\frac{c_i}{2}-1}{1+2c_f}$$

$$p_1=\frac{-4c_f}{1+2c_f} \qquad p_2=\frac{2c_f-1}{1+2c_f}$$

$$c_f=\frac{T_f}{T_0} \qquad c_i=\frac{T_0}{T_i} \qquad c_d=\frac{T_D}{T_0}$$

$$K_P=0.6K_{Pu} \qquad T_I=0.5T_u \qquad T_D=0.125T_u \qquad T_f=\frac{T_D}{\alpha}$$

Variables K_{Pu} and T_u are ultimate gain and ultimate period respectively.

Source code:

sid on-line identification s-function

zn2fd function for computation of controller parameters

Scqp controller s-function

See also:

On-line identification methods

Computation of ultimate gain and period
List of common controllers parameters

4.7 zn2tak

Takahashi's controller for processes of second order.

Transfer function of the controlled system:

$$G(z) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:
None.

Control law:

$$u_k = K_P\left[-y_k + y_{k-1} + \frac{T_0}{T_I}(w_k - y_k) + \frac{T_D}{T_0}(2y_{k-1} - y_k - y_{k-2})\right] + u_{k-1}$$

This form of control law can be transformed to feedback feedforward form:

$$u_k = r_0 w_k - q_0 y_k - q_1 y_{k-1} - q_2 y_{k-2} + u_{k-1}$$

where controller parameters r_0, q_0, q_1 and q_2 are calculated using following equation:

$$r_0 = K_P \frac{T_0}{T_I} \qquad q_0 = K_P\left(1 + \frac{T_0}{T_I} + \frac{T_D}{T_0}\right)$$

$$q_1 = -K_P\left(1 + 2\frac{T_D}{T_0}\right) \qquad q_2 = K_P \frac{T_D}{T_0}$$

$$K_P = 0.6K_{Pu}\left(1 - \frac{T_0}{T_u}\right) \qquad T_I = \frac{K_P T_u}{1.2K_{Pu}} \qquad T_D = \frac{3K_{Pu}T_u}{40K_p}$$

Variables K_{Pu} and T_u are ultimate gain and ultimate period respectively.

Source code:

sid on-line identification s-function

zn2tak function for computation of controller parameters

scrqp controller s-function

See also:
On-line identification methods
Computation of ultimate gain and period
List of common controllers parameters

4.8 zn2ast

Astrom's controller for second order processes.

Transfer function of the controlled system:

$$G(z) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:

Parameter	Description
alfa - filter constant	Filtration coefficient α used by filter of process output signal. The time constant of the filter is: $T_d = \frac{T_D}{\alpha}$ where usually 3 < α < 20
beta – weight	The weight β of reference signal in proportional component of controller. The weight should be 0 < $\beta \le$ 1

Control law:

$$u_k = u_{PI,k} + u_{D,k}$$

where:

$$u_{PI,k} = K_P\left[y_{k-1} - y_k + \frac{T_0}{2T_I}\left(w_k - y_k + w_{k-1} - y_{k-1}\right) + \beta\left(w_k - w_{k-1}\right)\right] + u_{PI,k-1}$$

$$u_{D,k} = \frac{T_D}{T_D + T_0\alpha}\left[K_P\alpha\left(y_{k-4} - y_k\right) + u_{D,k-1}\right]$$

where controller parameters K_p, T_0 and T_I are calculated using following equation:

$$K_P = 0.6K_{Pu} \qquad T_I = 0.5T_u \qquad T_D = 0.125T_u$$

Variables K_{Pu} and T_u are ultimate gain and ultimate period respectively.

Source code:

sid on-line identification s-function

zn2ast function for computation of controller parameters

scast controller s-function

See also:

On-line identification methods
Computation of ultimate gain and period
List of common controllers parameters

4.9 zn2fpd

Ziegler-Nichols controller for second order processes. The controller is based on forward rectangular method of discretization replacing derivation by a four-point difference.

Transfer function of the controlled system:

$$G(z)=\frac{b_1 z^{-1}+b_2 z^{-2}}{1+a_1 z^{-1}+a_2 z^{-2}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:
None.

Control law:

$$u_k = K_P\left[e_k - e_{k-1} + \frac{T_0}{T_I}e_k + \frac{T_D}{6T_0}\left(e_k + 2e_{k-1} - 6e_{k-2} + 2e_{k-3} + e_{k-4}\right)\right] + u_{k-1}$$

where e_k is control error ($e_k = w_k - y_k$). This form of control law can be transformed to feedback form:

$$u_k = q_0 e_k + q_1 e_{k-1} + q_2 e_{k-2} + q_3 e_{k-3} + q_4 e_{k-4} + u_{k-1}$$

Controller parameters are then calculated using following equations:

$$q_0 = K_P\left(1+\frac{T_0}{T_I}+\frac{T_D}{6T_0}\right) \qquad q_1 = K_P\left(-1+\frac{T_D}{3T_0}\right) \qquad q_2 = -K_P\frac{T_D}{T_0}$$

$$q_3 = K_P\frac{T_D}{3T_0} \qquad q_4 = K_P\frac{T_D}{6T_0}$$

$$K_P = 0.6K_{Pu} \qquad T_I = 0.5T_u \qquad T_D = 0.125T_u$$

Variables K_{Pu} and T_u are ultimate gain and ultimate period respectively.

Source code:

sid on-line identification s-function

zn2fpd function for computation of controller parameters

scfpd controller s-function

See also:
On-line identification methods
Computation of ultimate gain and period
List of common controllers parameters

4.10 ba2

Banyasz-Keviczky's controller for second order processes with dead time.

Transfer function of the controlled system

$$G(z)=\frac{b_1 z^{-1}+b_2 z^{-2}}{1+a_1 z^{-1}+a_2 z^{-2}} z^{-d}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:

Parameter	Description
ID Dead time	Dead time **d** of controlled process in sample times. Value of this parameter must be integer **d** ≥ 0.

Control law:

$$u_k = q_0 e_k + q_1 e_{k-1} + q_2 e_{k-2} - p_1 u_{k-1} - p_2 u_{k-2}$$

where e_k is control error ($e_k = w_k - y_k$) and controller parameters are calculated using following equations:

$$\gamma = \frac{\hat{b}_2}{\hat{b}_1} \qquad q_0 = \frac{k_I}{\hat{b}_1} \qquad q_1 = q_0 \hat{a}_1 \qquad q_2 = q_0 \hat{a}_2$$

and where:

- If $\gamma \le 0$ or $\gamma = 1$ use serial filter $G_F\left(z^{-1}\right) = \frac{1}{1+\gamma \cdot z^{-1}}$:

$$k_I = \frac{1}{2(d+1)-1} \qquad p_1 = -1+\gamma \qquad p_2 = -\gamma$$

- else do not use the filter:

$$k_I = \frac{1}{2(d+1)(1+\gamma)(1-\gamma)} \qquad p_1 = -1 \qquad p_2 = 0$$

Source code:

sid on-line identification s-function

ba2 function for computation of controller parameters

scqp controller s-function

See also:

On-line identification methods
List of common controllers parameters

4.11 da2

Dahlin's controller for the processes of the second order.

Transfer function of the controlled system:

$$G(z) = \frac{b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1, \hat{a}_2, \hat{b}_1\right]^{\mathrm{T}}$$

Controller specific parameters:

Parameter	Description
B – adjustment factor	Adjustment factor **B** that specifies dominant time constant of step response of closed control circuit: the smaller **B**, the quicker step response of closed circuit.

Control law:

$$u_k = K_P \left[e_k - e_{k-1} + \frac{T_0}{T_I} e_k + \frac{T_D}{T_0}\left(e_k - 2e_{k-1} + e_{k-2}\right) \right] + u_{k-1}$$

where e_k is control error ($e_k = w_k - y_k$) and controller parameters are calculated using following equations:

$$Q = 1 - e^{-\frac{T_0}{B}} \qquad K_P = -\frac{(\hat{a}_1 + 2\hat{a}_2)Q}{\hat{b}_1} \qquad T_D = \frac{T_0 \hat{a}_2 Q}{K_P \hat{b}_1} \qquad T_I = -\frac{T_0}{\frac{1}{\hat{a}_1 + 2\hat{a}_2} + 1 + \frac{T_D}{T_0}}$$

Source code:

sid on-line identification s-function

da2 function for computation of controller parameters

scqp controller s-function

See also:

On-line identification methods
List of common controllers parameters

4.12 pp2a_1

PID A-1 pole placement controller for second order processes.

Transfer function of the controlled system:

$$G(z)=\frac{b_1 z^{-1}+b_2 z^{-2}}{1+a_1 z^{-1}+a_2 z^{-2}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{b}_1,\hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:

Parameter	Description
xi – damping factor	Dumping factor ξ specifying dynamic behaviour of closed loop. The dynamic behaviour of the closed-loop is similar to second order continuous system with characteristic polynomial $s^2+2\cdot\xi\cdot\omega\cdot s+\omega^2$.
omega - natural frequency	Natural frequency ω specifying dynamic behaviour of closed loop. The dynamic behaviour of the closed-loop is similar to second order continuous system with characteristic polynomial $s^2+2\cdot\xi\cdot\omega\cdot s+\omega^2$.

Control law:

$$u_k=q_0e_k+q_1e_{k-1}+q_2e_{k-2}+(1-\gamma)u_{k-1}+\gamma\, u_{k-2}$$

where e_k is control error ($e_k=w_k-y_k$) and controller parameters q_0, q_1, q_2 and γ are calculated by solving following diophantine equation:

$$A\left(z^{-1}\right)P\left(z^{-1}\right)+B\left(z^{-1}\right)Q\left(z^{-1}\right)=D\left(z^{-1}\right)$$

where polynomials are as follows:

$$A\left(z^{-1}\right)=1+\hat{a}_1z^{-1}+\hat{a}_2z^{-2} \qquad B\left(z^{-1}\right)=\hat{b}_1z^{-1}+\hat{b}_2z^{-2}$$

$$P\left(z^{-1}\right)=\left(1-z^{-1}\right)\left(1+\gamma z^{-1}\right) \qquad Q\left(z^{-1}\right)=q_0+q_1z^{-1}+q_2z^{-2}$$

$$D\left(z^{-1}\right)=1+d_1z^{-1}+d_2z^{-2} \qquad d_1=-2\exp\left(-\xi\omega T_0\right)\cos\left(\omega T_0\sqrt{1-\xi^2}\right) \quad \text{for } \xi\le 1$$

$$d_1=-2\exp\left(-\xi\omega T_0\right)\cosh\left(\omega T_0\sqrt{\xi^2-1}\right) \quad \text{for } \xi>1$$

$$d_2=\exp\left(-2\xi\,\omega\,T_0\right)$$

Solving the diophantine equation leads to following relations for controller parameters:

$$\gamma=q_2\frac{\hat{b}_2}{\hat{a}_2} \qquad q_2=\frac{s_1}{r_1} \qquad q_1=\frac{\hat{a}_2}{\hat{b}_2}-q_2\left(\frac{\hat{b}_1}{\hat{b}_2}-\frac{\hat{a}_1}{\hat{a}_2}+1\right) \qquad q_0=\frac{1}{\hat{b}_1}\left(d_1+1-\hat{a}_1-\gamma\right)$$

where:

$$r_1=\left(\hat{b}_1+\hat{b}_2\right)\left(\hat{a}_1\hat{b}_1\hat{b}_2-\hat{a}_2\hat{b}_1^2-\hat{b}_2^2\right) \qquad s_1=\hat{a}_2\left[\left(\hat{b}_1+\hat{b}_2\right)\left(\hat{a}_1\hat{b}_2-\hat{a}_2\hat{b}_1\right)+\hat{b}_2\left(\hat{b}_1d_2-\hat{b}_2d_1-\hat{b}_2\right)\right]$$

Source code:

sid on-line identification s-function

pp2a_1 function for computation of controller parameters

scqp controller s-function

See also:

On-line identification methods
List of common controllers parameters

4.13 pp2a_2

PID A-2 pole placement controller for second order processes.

Transfer function of the controlled system:

$$G(z)=\frac{b_1 z^{-1}+b_2 z^{-2}}{1+a_1 z^{-1}+a_2 z^{-2}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:

Parameter	Description
omega – imaginary component of the pole	Imaginary component ω of the poles of the closed control loop. The dynamic behaviour of the closed-loop is defined by its poles: $z_1=\alpha+j\cdot\omega$ $z_2=\alpha-j\cdot\omega$ $z_3, z_4=\alpha$
alfa – real component of the pole	Real component α of the poles of the closed control loop. The dynamic behaviour of the closed-loop is defined by its poles: $z_1=\alpha+j\cdot\omega$ $z_2=\alpha-j\cdot\omega$ $z_3, z_4=\alpha$

Control law:

$$u_k=q_0 e_k+q_1 e_{k-1}+q_2 e_{k-2}+(1-\gamma)u_{k-1}+\gamma\, u_{k-2}$$

where e_k is control error ($e_k=w_k-y_k$) and controller parameters q_0, q_1, q_2 and γ are calculated by solving following diophantine equation:

$$A\left(z^{-1}\right)P\left(z^{-1}\right)+B\left(z^{-1}\right)Q\left(z^{-1}\right)=D\left(z^{-1}\right)$$

where polynomials are as follows:

$$A\left(z^{-1}\right)=1+\hat{a}_1 z^{-1}+\hat{a}_2 z^{-2} \qquad B\left(z^{-1}\right)=\hat{b}_1 z^{-1}+\hat{b}_2 z^{-2}$$

$$P\left(z^{-1}\right)=\left(1-z^{-1}\right)\left(1+\gamma z^{-1}\right) \qquad Q\left(z^{-1}\right)=q_0+q_1 z^{-1}+q_2 z^{-2}$$

$$D\left(z^{-1}\right)=\left[1-(\alpha+j\omega)z^{-1}\right]\cdot\left[1-(\alpha-j\omega)z^{-1}\right]\cdot\left(1-\alpha\, z^{-1}\right)^2$$

Solving the diophantine equation leads to following relations for controller parameters:

$$\gamma=\frac{r_6}{r_1} \qquad q_0=\frac{r_2-r_3}{r_1} \qquad q_1=-\frac{r_4+r_5}{r_1} \qquad q_2=\frac{x_4+\gamma\ a_2}{b_2}$$

where:

$$r_1=\left(\hat{b}_1+\hat{b}_2\right)\left(\hat{a}_1\hat{b}_1\hat{b}_2-\hat{a}_2\hat{b}_1^2-\hat{b}_2^2\right) \qquad r_2=x_1\left(\hat{b}_1+\hat{b}_2\right)\left(\hat{a}_1\hat{b}_2-\hat{a}_2\hat{b}_1\right)$$

$$r_3 = \hat{b}_1^2 x_4 - \hat{b}_2 \left[\hat{b}_1 x_3 - \hat{b}_2 \left(x_1 + x_2 \right) \right] \qquad r_4 = a_1 \left[\hat{b}_1^2 x_4 + \hat{b}_2^2 x_1 - \hat{b}_1 \hat{b}_2 \left(x_2 + x_3 \right) \right]$$

$$r_5 = \left(\hat{b}_1 + \hat{b}_2 \right) \left[\hat{a}_2 \left(\hat{b}_1 x_2 - \hat{b}_2 x_1 \right) - \hat{b}_1 x_4 + \hat{b}_2 x_3 \right] \qquad r_6 = \hat{b}_1 \left(\hat{b}_1^2 x_4 - \hat{b}_1 \hat{b}_2 x_3 + \hat{b}_2^2 x_2 \right) - \hat{b}_2^3 x_1$$

$$x_1 = c + 1 - \hat{a}_1 \qquad x_2 = d + \hat{a}_1 - \hat{a}_2 \qquad x_3 = f + \hat{a}_2 \qquad x_4 = g$$

$$c = -4\alpha \qquad d = 6\alpha^2 + \omega^2 \qquad f = -2\alpha \left(2\alpha^2 + \omega^2 \right) \qquad g = \alpha^2 \left(\alpha^2 + \omega^2 \right)$$

Source code:

sid on-line identification s-function

pp2a_2 function for computation of controller parameters

scqp controller s-function

See also:

On-line identification methods

List of common controllers parameters

4.14 pp2b_1

PID B-1 pole placement controller for second order processes.

Transfer function of the controlled system:

$$G(z)=\frac{b_1 z^{-1}+b_2 z^{-2}}{1+a_1 z^{-1}+a_2 z^{-2}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:

Parameter	Description
xi - damping factor	Dumping factor ξ specifying dynamic behaviour of closed loop. The dynamic behaviour of the closed-loop is similar to second order continuous system with characteristic polynomial $s^2+2\cdot\xi\cdot\omega\cdot s+\omega^2$.
omega - natural frequency	Natural frequency ω specifying dynamic behaviour of closed loop. The dynamic behaviour of the closed-loop is similar to second order continuous system with characteristic polynomial $s^2+2\cdot\xi\cdot\omega\cdot s+\omega^2$.

Control law:

$$u_k=-\left[\left(q_0'+\beta\right)y_k-\left(q_0'+q_2'\right)y_{k-1}+q_2' y_{k-2}\right]-(\gamma-1)u_{k-1}+\gamma u_{k-2}+\beta w_k$$

This form of control law can be transformed to feedback feedforward form:

$$u_k=r_0 w_k-q_0 y_k-q_1 y_{k-1}-q_2 y_{k-2}-p_1 u_{k-1}-p_2 u_{k-2}$$

where controller parameters are calculated by solving following diophantine equation:

$$A\left(z^{-1}\right)P\left(z^{-1}\right)+B\left(z^{-1}\right)\left[Q\left(z^{-1}\right)+r_0\right]=D\left(z^{-1}\right)$$

where polynomials are as follows:

$$A\left(z^{-1}\right)=1+\hat{a}_1 z^{-1}+\hat{a}_2 z^{-2} \qquad B\left(z^{-1}\right)=\hat{b}_1 z^{-1}+\hat{b}_2 z^{-2}$$

$$P\left(z^{-1}\right)=\left(1-z^{-1}\right)\left(1+\gamma z^{-1}\right) \qquad Q\left(z^{-1}\right)=\left(1-z^{-1}\right)\left(q_0'+q_2' z^{-1}\right)$$

$$D\left(z^{-1}\right)=1+d_1 z^{-1}+d_2 z^{-2} \qquad d_1=-2\exp\left(-\xi\omega T_0\right)\cos\left(\omega T_0\sqrt{1-\xi^2}\right) \quad \text{for } \xi\le 1$$

$$d_1=-2\exp\left(-\xi\omega T_0\right)\cosh\left(\omega T_0\sqrt{\xi^2-1}\right) \quad \text{for } \xi>1$$

$$d_2=\exp\left(-2\xi\,\omega\,T_0\right)$$

Solving the diophantine equation leads to following relations for controller parameters:

$$r_0=\frac{d_1+1-\hat{a}_1-\gamma-\hat{b}_1 q_2'}{\hat{b}_1} \qquad p_1=\gamma-1 \qquad p_2=-\gamma$$

$$q_0=r_0+q_0' \qquad q_1=-\left(q_o'+q_2'\right) \qquad q_2=q_2'$$

where:

$$q_2' = \frac{s_1}{r_1} \qquad q_0' = q_2'\left(\frac{\hat{b}_1}{\hat{b}_2} - \frac{\hat{a}_1}{\hat{a}_2}\right) - \frac{\hat{a}_2}{\hat{b}_2} \qquad \gamma = q_2' \frac{\hat{b}_2}{\hat{a}_2}$$

$$r_1 = \left(\hat{b}_1 + \hat{b}_2\right)\left(\hat{a}_1\hat{b}_1\hat{b}_2 - \hat{a}_2\hat{b}_1^2 - \hat{b}_2^2\right) \qquad s_1 = \hat{a}_2\left\{\hat{b}_2\left[\hat{a}_1\left(\hat{b}_1 + \hat{b}_2\right) + \hat{b}_1\left(d_2 - \hat{a}_2\right) - \hat{b}_2\left(d_1 + 1\right)\right] - \hat{a}_2\hat{b}_1^2\right\}$$

Source code:

sid on-line identification s-function

pp2b_1 function for computation of controller parameters

scrqp controller s-function

See also:

On-line identification methods
List of common controllers parameters

4.15 pp2b_2

PID B-2 pole placement controller for second order processes.

Transfer function of the controlled system:

$$G(z)=\frac{b_1 z^{-1}+b_2 z^{-2}}{1+a_1 z^{-1}+a_2 z^{-2}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:

Parameter	Description
omega – imaginary component of the pole	Imaginary component ω of the poles of the closed control loop. The dynamic behaviour of the closed-loop is defined by its poles: $z_1=\alpha+j\cdot\varpi$ $z_2=\alpha-j\cdot\varpi$ $z_3, z_4=\alpha$
alfa – real component of the pole	Real component α of the poles of the closed control loop. The dynamic behaviour of the closed-loop is defined by its poles: $z_1=\alpha+j\cdot\varpi$ $z_2=\alpha-j\cdot\varpi$ $z_3, z_4=\alpha$

Control law:

$$u_k=-\left[\left(q_0'+\beta\right)y_k-\left(q_0'+q_2'\right)y_{k-1}+q_2' y_{k-2}\right]-(\gamma-1)u_{k-1}+\gamma u_{k-2}+\beta w_k$$

This form of control law can be transformed to feedback feedforward form:

$$u_k=r_0 w_k-q_0 y_k-q_1 y_{k-1}-q_2 y_{k-2}-p_1 u_{k-1}-p_2 u_{k-2}$$

where controller parameters are calculated by solving following diophantine equation:

$$A\left(z^{-1}\right)P\left(z^{-1}\right)+B\left(z^{-1}\right)\left[Q\left(z^{-1}\right)+r_0\right]=D\left(z^{-1}\right)$$

where polynomials are as follows:

$$A\left(z^{-1}\right)=1+\hat{a}_1 z^{-1}+\hat{a}_2 z^{-2} \qquad B\left(z^{-1}\right)=\hat{b}_1 z^{-1}+\hat{b}_2 z^{-2}$$

$$P\left(z^{-1}\right)=\left(1-z^{-1}\right)\left(1+\gamma z^{-1}\right) \qquad Q\left(z^{-1}\right)=\left(1-z^{-1}\right)\left(q_0'+q_2' z^{-1}\right)$$

$$D\left(z^{-1}\right)=\left[1-(\alpha+j\omega)z^{-1}\right]\cdot\left[1-(\alpha-j\omega)z^{-1}\right]\cdot\left(1-\alpha\, z^{-1}\right)^2$$

Solving the diophantine equation leads to following relations for controller parameters:

$$r_0=\frac{x_1+x_2-x_3+x_4}{\hat{b}_1+\hat{b}_2} \qquad p_1=\gamma-1 \qquad p_2=-\gamma$$

$$q_0=r_0+q_0' \qquad q_1=-\left(q_o'+q_2'\right) \qquad q_2=q_2'$$

where:

$$\gamma = \frac{r_5}{r_1} \qquad q_0' = -\frac{r_2 + r_3 + r_4}{r_1} \qquad q_2' = \frac{r_6 + r_7}{r_1}$$

$$r_1 = \left(\hat{b}_1 + \hat{b}_2\right)\left(\hat{a}_1\hat{b}_1\hat{b}_2 - \hat{a}_2\hat{b}_1^2 - \hat{b}_2^2\right) \qquad r_2 = \hat{a}_1\hat{b}_2\left[\hat{b}_1\left(x_2 - x_3 + x_4\right) - \hat{b}_2 x_1\right]$$

$$r_3 = \hat{a}_2\hat{b}_1\left[\hat{b}_2 x_1 - \hat{b}_1\left(x_2 - x_3 + x_4\right)\right] \qquad r_4 = \left(\hat{b}_1 + \hat{b}_2\right)\left[\hat{b}_1 x_4 + \hat{b}_2\left(x_3 - x_4\right)\right]$$

$$r_5 = \hat{b}_1\left(\hat{b}_1^2 x_4 + \hat{b}_1\hat{b}_2 x_3 + \hat{b}_2^2 x_2\right) - \hat{b}_2^3 x_1 \qquad r_6 = \hat{b}_1^2\left(\hat{a}_2 x_3 + \hat{a}_1 x_4 - \hat{a}_2 x_4\right)$$

$$r_7 = \hat{b}_2\left[\hat{b}_1\left(\hat{a}_1 x_4 + \hat{a}_2 x_2 - x_4\right) - \hat{b}_2\left(\hat{a}_2 x_1 + x_4\right)\right]$$

$$x_1 = c + 1 - \hat{a}_1 \qquad x_2 = d + \hat{a}_1 - \hat{a}_2 \qquad x_3 = -f - \hat{a}_2 \qquad x_4 = g$$

$$c = -4\alpha \qquad d = 6\alpha^2 + \omega^2 \qquad f = -2\alpha\left(2\alpha^2 + \omega^2\right) \qquad g = \alpha^2\left(\alpha^2 + \omega^2\right)$$

Source code:

sid on-line identification s-function

pp2b_2 function for computation of controller parameters

scrqp controller s-function

See also:

On-line identification methods

List of common controllers parameters

4.16 db2w

Deadbeat controller for processes of second order (weak version).

Transfer function of the controlled system:

$$G(z)=\frac{b_1 z^{-1}+b_2 z^{-2}}{1+a_1 z^{-1}+a_2 z^{-2}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{b}_1,\hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:
None.

Control law:

$$u_k=r_0 w_k-q_0 y_k-q_1 y_{k-1}-p_1 u_{k-1}$$

where controller parameters r_0, q_0, q_1 and p_1 are calculated using following equation:

$$r_0=\frac{1}{\hat{b}_1} \qquad q_0=-\frac{\hat{a}_1}{\hat{b}_1} \qquad q_1=-\frac{\hat{a}_2}{\hat{b}_1} \qquad p_1=\frac{\hat{b}_2}{\hat{b}_1}$$

Source code:

sid on-line identification s-function

db2w function for computation of controller parameters

scrqp controller s-function

See also:
On-line identification methods
List of common controllers parameters

4.17 db2s

Deadbeat controller for processes of second order and step changes of reference variable (strong version).

Transfer function of the controlled system:

$$G(z) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:
None.

Control law:

$$u_k = r_0 w_k - q_0 y_k - q_1 y_{k-1} - p_1 u_{k-1}$$

where controller parameters r_0, q_0, q_1 and p_1 are calculated by solving following diophantine equation:

$$A\left(z^{-1}\right)P\left(z^{-1}\right) + B\left(z^{-1}\right)Q\left(z^{-1}\right) = 1$$

$$1 - B\left(z^{-1}\right)R\left(z^{-1}\right) = \left(1 - z^{-1}\right)S\left(z^{-1}\right)$$

where polynomials are as follows:

$$A\left(z^{-1}\right) = 1 + \hat{a}_1 z^{-1} + \hat{a}_2 z^{-2} \qquad B\left(z^{-1}\right) = \hat{b}_1 z^{-1} + \hat{b}_2 z^{-2}$$

$$P\left(z^{-1}\right) = 1 + p_1 z^{-1} \qquad Q\left(z^{-1}\right) = q_0 + q_1 z^{-1}$$

$$R\left(z^{-1}\right) = r_0 \qquad S\left(z^{-1}\right) = s_0 + s_1 z^{-1}$$

Solving the diophantine equations leads to following relations for controller parameters:

$$q_0 = \frac{-\hat{a}_1^2 \hat{b}_2 + \hat{a}_1 \hat{a}_2 \hat{b}_1 + \hat{a}_2 \hat{b}_1}{-\hat{b}_2^2 + \hat{a}_1 \hat{b}_1 \hat{b}_2 - \hat{a}_2 \hat{b}_1^2} \qquad q_1 = \frac{-\hat{a}_2\left(\hat{a}_1 \hat{b}_2 - \hat{a}_2 \hat{b}_1\right)}{-\hat{b}_2^2 + \hat{a}_1 \hat{b}_1 \hat{b}_2 - \hat{a}_2 \hat{b}_1^2}$$

$$p_1 = \frac{\hat{b}_2\left(\hat{a}_1 \hat{b}_2 - \hat{a}_2 \hat{b}_1\right)}{-\hat{b}_2^2 + \hat{a}_1 \hat{b}_1 \hat{b}_2 - \hat{a}_2 \hat{b}_1^2} \qquad r_0 = \frac{1}{\hat{b}_1 + \hat{b}_2}$$

Source code:

sid	on-line identification s-function
db2s	function for computation of controller parameters
scrqp	controller s-function

See also:
On-line identification methods
List of common controllers parameters

4.18 mv2

Minimum variance controller for processes of second order.

Transfer function of the controlled system:

$$G(z)=\frac{b_1 z^{-1}+b_2 z^{-2}}{1+a_1 z^{-1}+a_2 z^{-2}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{b}_1,\hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:

Parameter	Description
q - penalisation factor	Penalisation factor **q** used by computation of control law parameters. This parameter specifies the measure of change of current controller output with respect to previous controller output: the smaller penalisation, the greater possible change of controller output.

Control law:

$$u_k=\frac{1}{q}\left(\hat{a}_1 y_{k-1}+\hat{a}_2 y_{k-2}-\hat{b}_1 u_{k-1}-\hat{b}_2 u_{k-2}+w_k\right)+u_{k-1}$$

This form of control law can be transformed to feedback feedforward form:

$$u_k=r_0 w_k-q_1 y_{k-1}-q_2 y_{k-2}-p_1 u_{k-1}-p_2 u_{k-2}$$

where controller parameters r_0, q_1, q_2, p_1, and p_2 are calculated using following equation:

$$r_0=\frac{1}{q} \qquad q_1=-\frac{\hat{a}_1}{q} \qquad q_2=-\frac{\hat{a}_2}{q} \qquad p_1=\frac{\hat{b}_1}{q}-1 \qquad p_2=\frac{\hat{b}_2}{q}$$

Source code:

sid on-line identification s-function

mv2 function for computation of controller parameters

scrqp controller s-function

See also:

On-line identification methods
List of common controllers parameters

4.19 pp2chp

Generic pole placement controller for second order processes.

Transfer function of the controlled system:

$$G(z)=\frac{b_1 z^{-1}+b_2 z^{-2}}{1+a_1 z^{-1}+a_2 z^{-2}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{b}_1,\hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:

Parameter	Description
Reference signal type	It's possible to choose one of the following signal types: • steps • ramps • sin waves
Frequency	Frequency *f* of reference signal - valid only when reference signal is 'sin waves'
Coefficients of characteristic polynomial	Vector determining characteristic polynomial $D(z^{-1})$ and thus the positions of the poles of the closed loop. If vector is [*d0, d1, d2, d3*], then characteristic polynomial will be $D(z^{-1}) = d0 + d1 \cdot z^{-1} + d2 \cdot z^{-2} + d3 \cdot z^{-3}$. The length of the vector can vary but at lest *d0* must be specified.

Control law:

$$u_k=\frac{1}{p_0}\left(r_0 w_k+r_1 w_{k-1}-q_0 y_k-q_1 y_{k-1}-p_1 u_{k-1}\right)$$

where controller parameters r_0, r_1, q_0, q_1, p_0 and p_1 are calculated by solving following diophantine equation:

$$A\left(z^{-1}\right)P\left(z^{-1}\right)+B\left(z^{-1}\right)Q\left(z^{-1}\right)=D\left(z^{-1}\right)$$

$$B\left(z^{-1}\right)R\left(z^{-1}\right)+F\left(z^{-1}\right)S\left(z^{-1}\right)=D\left(z^{-1}\right)$$

where polynomials are as follows:

$A\left(z^{-1}\right)=1+\hat{a}_1 z^{-1}+\hat{a}_2 z^{-2}$ $\qquad$ $B\left(z^{-1}\right)=\hat{b}_1 z^{-1}+\hat{b}_2 z^{-2}$

$P\left(z^{-1}\right)=p_0+p_1 z^{-1}$ $\qquad$ $Q\left(z^{-1}\right)=q_0+q_1 z^{-1}$

$R\left(z^{-1}\right)=r_0+r_1 z^{-1}$ $\qquad$ $S\left(z^{-1}\right)$ is any appropriate polynomial

$F(z^{-1})$ depends on reference signal type:

Reference signal type	$F(z^{-1})$
steps	$1-z^{-1}$
ramps	$1-2z^{-1}+z^{-2}$
sin waves	$1-2\cdot\cos\omega\cdot z^{-1}+z^{-2}$ where $\omega=2\pi f\cdot T_0$

Source code:

sid on-line identification s-function

pp2chp function for computation of controller parameters

scast controller s-function

See also:

On-line identification methods

List of common controllers parameters

4.20 pp2lq

LQ controller for second order processes.

Transfer function of the controlled system:

$$G(z)=\frac{b_1z^{-1}+b_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{b}_1,\hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:

Parameter	Description
Reference signal type	It's possible to choose one of the following signal types: • steps • ramps • sin waves
Frequency	Frequency *f* of reference signal - valid only when reference signal is 'sin waves'
Penalization of controller output – fi	Penalization *φ* of controller output in LQ criterion: $J=\sum_{k=0}^{\infty}\left\{\left[w(k)-y(k)\right]^2+\varphi\left[u(k)\right]^2\right\}$

Control law:

$$u_k=\frac{1}{p_0}\left(r_0w_k+r_1w_{k-1}-q_0y_k-q_1y_{k-1}-p_1u_{k-1}\right)$$

where controller parameters r_0, r_1, q_0, q_1, p_0 and p_1 are calculated by solving following diophantine equation:

$$A\left(z^{-1}\right)P\left(z^{-1}\right)+B\left(z^{-1}\right)Q\left(z^{-1}\right)=D\left(z^{-1}\right)$$

$$B\left(z^{-1}\right)R\left(z^{-1}\right)+F\left(z^{-1}\right)S\left(z^{-1}\right)=D\left(z^{-1}\right)$$

where polynomials are as follows:

$A\left(z^{-1}\right)=1+\hat{a}_1z^{-1}+\hat{a}_2z^{-2}$ $\quad$ $B\left(z^{-1}\right)=\hat{b}_1z^{-1}+\hat{b}_2z^{-2}$

$P\left(z^{-1}\right)=p_0+p_1z^{-1}$ $\quad$ $Q\left(z^{-1}\right)=q_0+q_1z^{-1}$

$R\left(z^{-1}\right)=r_0+r_1z^{-1}$ $\quad$ $S\left(z^{-1}\right)$ is any appropriate polynomial

$D\left(z^{-1}\right)=1+d_1z^{-1}+d_2z^{-2}$

where:

$$d_1=\frac{m_1}{\delta+m_2},\ d_2=\frac{m_2}{\delta},\ \delta=\frac{\lambda+\sqrt{\lambda^2-4m_2^{\,2}}}{2},\ \lambda=\frac{m_0}{2}-m_2+\sqrt{\left(\frac{m_0}{2}+m_2\right)^2-m_1^{\,2}},$$

$$m_0=\varphi(1+a_1^{\,2}+a_2^{\,2})+b_1^{\,2}+b_2^{\,2},\ m_1=\varphi(a_1+a_1a_2)+b_1b_2,\ m_2=\varphi a_2$$

$F(z^{-1})$ depends on reference signal type:

Reference signal type	$F(z^{-1})$
steps	$1-z^{-1}$
ramps	$1-2z^{-1}+z^{-2}$
sin waves	$1-2\cdot\cos\omega\cdot z^{-1}+z^{-2}$ where $\omega=2\pi f\cdot T_0$

Source code:

sid on-line identification s-function

pp2lq function for computation of controller parameters

scast controller s-function

See also:

On-line identification methods
List of common controllers parameters

4.21 pp2c2dof

Pole placement 2 degree-of-freedom (2DOF) controller with compensator for second order processes.

Transfer function of the controlled system:

$$G(z)=\frac{b_1 z^{-1}+b_2 z^{-2}}{1+a_1 z^{-1}+a_2 z^{-2}}$$

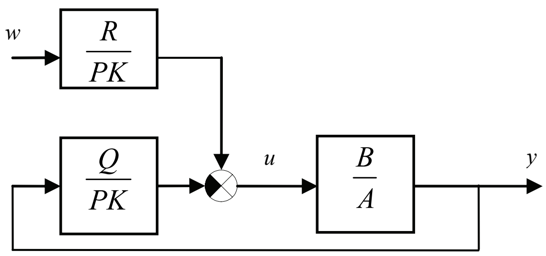
Vector of identification initial parameter estimations:

$$\left[\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2\right]^{\mathrm{T}}$$

Controller specific parameters:

Parameter	Description
xi – damping factor	Dumping factor ξ specifying dynamic behaviour of closed loop. The dynamic behaviour of the closed-loop is similar to second order continuous system with characteristic polynomial $s^2+2\cdot\xi\cdot\omega\cdot s+\omega^2$.
omega - natural frequency	Natural frequency ω specifying dynamic behaviour of closed loop. The dynamic behaviour of the closed-loop is similar to second order continuous system with characteristic polynomial $s^2+2\cdot\xi\cdot\omega\cdot s+\omega^2$.

2DOF control loop:



Feedback controller:

$$G_R(z)=\frac{Q\left(z^{-1}\right)}{P\left(z^{-1}\right)K\left(z^{-1}\right)}=\frac{q_0+q_1 z^{-1}+q_2 z^{-2}}{\left(1+p_1 z^{-1}\right)\left(1-z^{-1}\right)}$$

Feedforward controller for a step reference signal:

$$G_F(z)=\frac{R\left(z^{-1}\right)}{P\left(z^{-1}\right)K\left(z^{-1}\right)}=\frac{r_0}{\left(1+p_1 z^{-1}\right)\left(1-z^{-1}\right)}$$

Characteristic polynomial of closed loop:

$$A\left(z^{-1}\right)P\left(z^{-1}\right)K\left(z^{-1}\right)+B\left(z^{-1}\right)Q\left(z^{-1}\right)=D\left(z^{-1}\right)$$

where polynomials are as follows:

$$A\left(z^{-1}\right)=1+\hat{a}_1 z^{-1}+\hat{a}_2 z^{-2} \qquad B\left(z^{-1}\right)=\hat{b}_1 z^{-1}+\hat{b}_2 z^{-2}$$

$$P\left(z^{-1}\right)=1+p_1 z^{-1} \qquad Q\left(z^{-1}\right)=q_0+q_1 z^{-1}+q_2 z^{-2}$$

$$K\left(z^{-1}\right)=1-z^{-1}$$

$$D\left(z^{-1}\right)=1+d_1z^{-1}+\ldots+d_4z^{-4} \qquad d_1=-2\exp\left(-\xi\omega T_0\right)\cos\left(\omega T_0\sqrt{1-\xi^2}\right) \qquad \text{for } \xi\le 1$$

$$d_1=-2\exp\left(-\xi\omega T_0\right)\cosh\left(\omega T_0\sqrt{\xi^2-1}\right) \qquad \text{for } \xi>1$$

$$d_2=\exp\left(-2\xi\,\omega\,T_0\right)$$

$$d_3=d_4=0$$

Matrix equation:

$$\begin{bmatrix} \hat{b}_1 & 0 & 0 & 1 \\ \hat{b}_2 & \hat{b}_1 & 0 & \hat{a}_1-1 \\ 0 & \hat{b}_2 & \hat{b}_1 & \hat{a}_2-\hat{a}_1 \\ 0 & 0 & \hat{b}_2 & -\hat{a}_2 \end{bmatrix}\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ p_1 \end{bmatrix}=\begin{bmatrix} d_1+1-\hat{a}_1 \\ d_2+\hat{a}_1-\hat{a}_2 \\ d_3+\hat{a}_2 \\ d_4 \end{bmatrix}$$

Control law:

$$P(z^{-1})K(z^{-1})u_k=R(z^{-1})w_k-Q(z^{-1})y_k$$

$$u_k=r_0w_k-q_0y_k-q_1y_{k-1}-q_2y_{k-2}+\left(1-p_1\right)u_{k-1}+p_1u_{k-2}$$

$$r_0=\frac{1+d_1+d_2+d_3+d_4}{\hat{b}_1+\hat{b}_2}$$

Source code:

sid on-line identification s-function

pp2c2dof function for computation of controller parameters

scfbfw controller s-function

See also:

On-line identification methods
List of common controllers parameters

4.22 zn3fr

Ziegler-Nichols PID controller for processes of third order. The controller is based on forward rectangular method of discretization.

Transfer function of the controlled system:

$$G(z)=\frac{b_1z^{-1}+b_2z^{-2}+b_3z^{-3}}{1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{a}_3,\hat{b}_1,\hat{b}_2,\hat{b}_3\right]^{\mathrm{T}}$$

Controller specific parameters:

None.

Control law:

$$u_k=K_P\left[e_k-e_{k-1}+\frac{T_0}{T_I}e_{k-1}+\frac{T_D}{T_0}\left(e_k-2e_{k-1}+e_{k-2}\right)\right]+u_{k-1}$$

where e_k is control error ($e_k=w_k-y_k$). This form of control law can be transformed to feedback form:

$$u_k=q_0e_k+q_1e_{k-1}+q_2e_{k-2}+u_{k-1}$$

Controller parameters are calculated using following equations:

$$q_0=K_P\left(1+\frac{T_D}{T_0}\right)\qquad q_1=-K_P\left(1-\frac{T_0}{T_I}+2\frac{T_D}{T_0}\right)\qquad q_2=K_P\frac{T_D}{T_0}$$

$$K_P=0.6K_{Pu}\qquad T_I=0.5T_u\qquad T_D=0.125T_u$$

Variables K_{Pu} and T_u are ultimate gain and ultimate period respectively.

Source code:

sid on-line identification s-function

zn3fr function for computation of controller parameters

scqp controller s-function

See also:

On-line identification methods
Computation of ultimate gain and period
List of common controllers parameters

4.23 zn3br

Ziegler-Nichols PID controller for processes of third order. The controller is based on backward rectangular method of discretization.

Transfer function of the controlled system:

$$G(z)=\frac{b_1z^{-1}+b_2z^{-2}+b_3z^{-3}}{1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{a}_3,\hat{b}_1,\hat{b}_2,\hat{b}_3\right]^{\mathrm{T}}$$

Controller specific parameters:

None.

Control law:

$$u_k=K_P\left[e_k-e_{k-1}+\frac{T_0}{T_I}e_k+\frac{T_D}{T_0}\left(e_k-2e_{k-1}+e_{k-2}\right)\right]+u_{k-1}$$

where e_k is control error ($e_k=w_k-y_k$). This form of control law can be transformed to feedback form:

$$u_k=q_0e_k+q_1e_{k-1}+q_2e_{k-2}+u_{k-1}$$

Controller parameters are calculated using following equations:

$$q_0=K_P\left(1+\frac{T_0}{T_I}+\frac{T_D}{T_0}\right) \qquad q_1=-K_P\left(1+2\frac{T_D}{T_0}\right) \qquad q_2=K_P\frac{T_D}{T_0}$$

$$K_P=0.6K_{Pu} \qquad T_I=0.5T_u \qquad T_D=0.125T_u$$

Variables K_{Pu} and T_u are ultimate gain and ultimate period respectively.

Source code:

sid on-line identification s-function

zn3br function for computation of controller parameters

scqp controller s-function

See also:

On-line identification methods

Computation of ultimate gain and period

List of common controllers parameters

4.24 zn3tr

Ziegler-Nichols PID controller for processes of third order. The controller is based on trapezoidal method of discretization.

Transfer function of the controlled system:

$$G(z)=\frac{b_1 z^{-1}+b_2 z^{-2}+b_3 z^{-3}}{1+a_1 z^{-1}+a_2 z^{-2}+a_3 z^{-3}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{a}_3,\hat{b}_1,\hat{b}_2,\hat{b}_3\right]^{\mathrm{T}}$$

Controller specific parameters:
None.

Control law:

$$u_k=K_P\left[e_k-e_{k-1}+\frac{T_0}{T_I}\frac{e_k-e_{k-1}}{2}+\frac{T_D}{T_0}\left(e_k-2e_{k-1}+e_{k-2}\right)\right]+u_{k-1}$$

where e_k is control error ($e_k=w_k-y_k$). This form of control law can be transformed to feedback form:

$$u_k=q_0e_k+q_1e_{k-1}+q_2e_{k-2}+u_{k-1}$$

Controller parameters are calculated using following equations:

$$q_0=K_P\left(1+\frac{T_0}{2T_I}+\frac{T_D}{T_0}\right) \qquad q_1=-K_P\left(1-\frac{T_0}{2T_I}+\frac{2T_D}{T_0}\right) \qquad q_2=K_P\frac{T_D}{T_0}$$

$$K_P=0.6K_{Pu} \qquad T_I=0.5T_u \qquad T_D=0.125T_u$$

Variables K_{Pu} and T_u are ultimate gain and ultimate period respectively.

Source code:

sid on-line identification s-function

zn3tr function for computation of controller parameters

scqp controller s-function

See also:
On-line identification methods
Computation of ultimate gain and period
List of common controllers parameters

4.25 zn3pd

Ziegler-Nichols PD controller for processes of third order. The controller uses reference variable (w) just in proportional component.

Transfer function of the controlled system:

$$G(z)=\frac{b_1z^{-1}+b_2z^{-2}+b_3z^{-3}}{1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{a}_3,\hat{b}_1,\hat{b}_2,\hat{b}_3\right]^{\mathrm{T}}$$

Controller specific parameters:
None.

Control law:

$$u_k=K_P\left[w_K-y_k+\frac{T_D}{T_0}\left(y_{k-1}-y_k\right)\right]$$

This form of control law can be transformed to feedback feedforward form:

$$u_k=r_0w_k-q_0y_k-q_1y_{k-1}$$

where controller parameters r_0, q_0 and q_1 are calculated using following equation:

$$r_0=K_P \qquad q_0=K_P\left(1+\frac{T_D}{T_0}\right) \qquad q_1=-K_P\frac{T_D}{T_0}$$

$$K_P=0.4K_{Pu} \qquad T_D=\frac{T_u}{20}$$

Variables K_{Pu} and T_u are ultimate gain and ultimate period respectively.

Source code:

sid on-line identification s-function

zn3pd function for computation of controller parameters

scrqp controller s-function

See also:
On-line identification methods
Computation of ultimate gain and period
List of common controllers parameters

4.26 zn3pi

Ziegler-Nichols PI controller for processes of third order. The controller is based on trapezoidal method of discretization.

Transfer function of the controlled system:

$$G(z)=\frac{b_1 z^{-1}+b_2 z^{-2}+b_3 z^{-3}}{1+a_1 z^{-1}+a_2 z^{-2}+a_3 z^{-3}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{a}_3,\hat{b}_1,\hat{b}_2,\hat{b}_3\right]^{\mathrm{T}}$$

Controller specific parameters:
None.

Control law:

$$u_k=K_P\left[e_k-e_{k-1}+\frac{T_0}{T_I}\frac{e_k-e_{k-1}}{2}\right]+u_{k-1}$$

where e_k is control error ($e_k=w_k-y_k$). This form of control law can be transformed to feedback form:

$$u_k=q_0 e_k+q_1 e_{k-1}+u_{k-1}$$

Controller parameters are calculated using following equations:

$$q_0=K_P\left(1+\frac{T_0}{2T_I}\right) \qquad q_1=-K_P\left(1-\frac{T_0}{2T_I}\right)$$

$$K_P=0.6K_{Pu} \qquad T_I=0.5T_u$$

Variables K_{Pu} and T_u are ultimate gain and ultimate period respectively.

Source code:

sid on-line identification s-function

zn3pi function for computation of controller parameters

scqp controller s-function

See also:
On-line identification methods
Computation of ultimate gain and period
List of common controllers parameters

4.27 zn3fd

Controller for third order processes with filtration of D-component using Tustin approximation.

Transfer function of the controlled system:

$$G(z)=\frac{b_1z^{-1}+b_2z^{-2}+b_3z^{-3}}{1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{a}_3,\hat{b}_1,\hat{b}_2,\hat{b}_3\right]^{\mathrm{T}}$$

Controller specific parameters:

Parameter	Description
alfa - filtration coefficient	Filtration coefficient α used by filter of process output signal. The time constant of the filter is: $T_d=\frac{T_D}{\alpha}$ where usually $3<\alpha<20$

Control law:

$$u_k=q_0e_k+q_1e_{k-1}+q_2e_{k-2}-p_1u_{k-1}-p_2u_{k-2}$$

where e_k is control error ($e_k=w_k-y_k$) and controller parameters are calculated using following equations:

$$q_0=K_P\frac{1+2\left(c_f+c_d\right)+\frac{c_i}{2}\left(1+2c_f\right)}{1+2c_f} \qquad q_1=K_P\frac{\frac{c_i}{2}-4\left(c_f+c_d\right)}{1+2c_f}$$

$$q_2=K_P\frac{c_f\left(2-c_i\right)+2c_d+\frac{c_i}{2}-1}{1+2c_f}$$

$$p_1=\frac{-4c_f}{1+2c_f} \qquad p_2=\frac{2c_f-1}{1+2c_f}$$

$$c_f=\frac{T_f}{T_0} \qquad c_i=\frac{T_0}{T_i} \qquad c_d=\frac{T_D}{T_0}$$

$$K_P=0.6K_{Pu} \qquad T_I=0.5T_u \qquad T_D=0.125T_u \qquad T_f=\frac{T_D}{\alpha}$$

Variables K_{Pu} and T_u are ultimate gain and ultimate period respectively.

Source code:

sid on-line identification s-function

zn3fd function for computation of controller parameters

scqp controller s-function

See also:

On-line identification methods
Computation of ultimate gain and period

List of common controllers parameters

4.28 zn3tak

Takahashi's controller for processes of third order.

Transfer function of the controlled system:

$$G(z)=\frac{b_1 z^{-1}+b_2 z^{-2}+b_3 z^{-3}}{1+a_1 z^{-1}+a_2 z^{-2}+a_3 z^{-3}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{b}_1, \hat{b}_2, \hat{b}_3\right]^{\mathrm{T}}$$

Controller specific parameters:
None.

Control law:

$$u_k = K_P\left[-y_k + y_{k-1} + \frac{T_0}{T_I}\left(w_k - y_k\right) + \frac{T_D}{T_0}\left(2y_{k-1} - y_k - y_{k-2}\right)\right] + u_{k-1}$$

This form of control law can be transformed to feedback feedforward form:

$$u_k = r_0 w_k - q_0 y_k - q_1 y_{k-1} - q_2 y_{k-2} + u_{k-1}$$

where controller parameters r_0, q_0, q_1 and q_2 are calculated using following equation:

$$r_0 = K_P \frac{T_0}{T_I} \qquad q_0 = K_P\left(1+\frac{T_0}{T_I}+\frac{T_D}{T_0}\right)$$

$$q_1 = -K_P\left(1+2\frac{T_D}{T_0}\right) \qquad q_2 = K_P \frac{T_D}{T_0}$$

$$K_P = 0.6K_{Pu}\left(1-\frac{T_0}{T_u}\right) \qquad T_I = \frac{K_P T_u}{1.2K_{Pu}} \qquad T_D = \frac{3K_{Pu}T_u}{40K_p}$$

Variables K_{Pu} and T_u are ultimate gain and ultimate period respectively.

Source code:

sid on-line identification s-function

zn3tak function for computation of controller parameters

scrqp controller s-function

See also:
On-line identification methods
Computation of ultimate gain and period
List of common controllers parameters

4.29 zn3ast

Astrom's controller for third order processes.

Transfer function of the controlled system:

$$G(z)=\frac{b_1z^{-1}+b_2z^{-2}+b_3z^{-3}}{1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{a}_3,\hat{b}_1,\hat{b}_2,\hat{b}_3\right]^{\mathrm{T}}$$

Controller specific parameters:

Parameter	Description
alfa - filter constant	Filtration coefficient α used by filter of process output signal. The time constant of the filter is: $T_d=\frac{T_D}{\alpha}$ where usually $3 < \alpha < 20$
beta – weight	The weight β of reference signal in proportional component of controller. The weight should be $0 < \beta \le 1$

Control law:

$$u_k=u_{PI,k}+u_{D,k}$$

where:

$$u_{PI,k}=K_P\left[y_{k-1}-y_k+\frac{T_0}{2T_I}\left(w_k-y_k+w_{k-1}-y_{k-1}\right)+\beta\left(w_k-w_{k-1}\right)\right]+u_{PI,k-1}$$

$$u_{D,k}=\frac{T_D}{T_D+T_0\alpha}\left[K_P\alpha\left(y_{k-4}-y_k\right)+u_{D,k-1}\right]$$

where controller parameters K_p, T_0 and T_I are calculated using following equation:

$$K_P=0.6K_{Pu} \qquad T_I=0.5T_u \qquad T_D=0.125T_u$$

Variables K_{Pu} and T_u are ultimate gain and ultimate period respectively.

Source code:

sid on-line identification s-function

zn3ast function for computation of controller parameters

scast controller s-function

See also:

On-line identification methods
Computation of ultimate gain and period
List of common controllers parameters

4.30 zn3fpd

Ziegler-Nichols controller for third order processes. The controller is based on forward rectangular method of discretization replacing derivation by a four-point difference.

Transfer function of the controlled system:

$$G(z)=\frac{b_1z^{-1}+b_2z^{-2}+b_3z^{-3}}{1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{a}_3,\hat{b}_1,\hat{b}_2,\hat{b}_3\right]^{\mathrm{T}}$$

Controller specific parameters:

None.

Control law:

$$u_k=K_P\left[e_k-e_{k-1}+\frac{T_0}{T_I}e_k+\frac{T_D}{6T_0}\left(e_k+2e_{k-1}-6e_{k-2}+2e_{k-3}+e_{k-4}\right)\right]+u_{k-1}$$

where e_k is control error ($e_k=w_k-y_k$). This form of control law can be transformed to feedback form:

$$u_k=q_0e_k+q_1e_{k-1}+q_2e_{k-2}+q_3e_{k-3}+q_4e_{k-4}+u_{k-1}$$

Controller parameters are then calculated using following equations:

$$q_0=K_P\left(1+\frac{T_0}{T_I}+\frac{T_D}{6T_0}\right)\qquad q_1=K_P\left(-1+\frac{T_D}{3T_0}\right)\qquad q_2=-K_P\frac{T_D}{T_0}$$

$$q_3=K_P\frac{T_D}{3T_0}\qquad q_4=K_P\frac{T_D}{6T_0}$$

$$K_P=0.6K_{Pu}\qquad T_I=0.5T_u\qquad T_D=0.125T_u$$

Variables K_{Pu} and T_u are ultimate gain and ultimate period respectively.

Source code:

sid	on-line identification s-function
zn3fpd	function for computation of controller parameters
scfpd	controller s-function

See also:

On-line identification methods
Computation of ultimate gain and period
List of common controllers parameters

4.31 db3w

Deadbeat controller for processes of third order (weak version).

Transfer function of the controlled system:

$$G(z)=\frac{b_1z^{-1}+b_2z^{-2}+b_3z^{-3}}{1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{a}_3,\hat{b}_1,\hat{b}_2,\hat{b}_3\right]^{\mathrm{T}}$$

Controller specific parameters:
None.

Control law:

$$u_k=r_0w_k-q_0y_k-q_1y_{k-1}-q_2y_{k-2}-p_1u_{k-1}-p_2y_{k-2}$$

where controller parameters r_0, q_0, q_1, q_2, p_1 and p_2 are calculated using following equation:

$$r_0=\frac{1}{\hat{b}_1}\qquad q_0=-\frac{\hat{a}_1}{\hat{b}_1}\qquad q_1=-\frac{\hat{a}_2}{\hat{b}_1}\qquad q_2=-\frac{\hat{a}_3}{\hat{b}_1}\qquad p_1=\frac{\hat{b}_2}{\hat{b}_1}\qquad p_2=\frac{\hat{b}_3}{\hat{b}_1}$$

Source code:

sid on-line identification s-function

db3w function for computation of controller parameters

scrqp controller s-function

See also:
On-line identification methods
List of common controllers parameters

4.32 db3s

Deadbeat controller for processes of third order and step changes of reference variable (strong version).

Transfer function of the controlled system:

$$G(z)=\frac{b_1z^{-1}+b_2z^{-2}+b_3z^{-3}}{1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{a}_3,\hat{b}_1,\hat{b}_2,\hat{b}_3\right]^{\mathrm{T}}$$

Controller specific parameters:
None.

Control law:

$$u_k=r_0w_k-q_0y_k-q_1y_{k-1}-q_2y_{k-2}-p_1u_{k-1}-p_2y_{k-2}$$

where controller parameters r_0, q_0, q_1, q_2, p_1 and p_2 are calculated by solving following diophantine equation:

$$A\left(z^{-1}\right)P\left(z^{-1}\right)+B\left(z^{-1}\right)Q\left(z^{-1}\right)=1$$

$$1-B\left(z^{-1}\right)R\left(z^{-1}\right)=\left(1-z^{-1}\right)S\left(z^{-1}\right)$$

where polynomials are as follows:

$$A\left(z^{-1}\right)=1+\hat{a}_1z^{-1}+\hat{a}_2z^{-2}+\hat{a}_3z^{-3} \qquad B\left(z^{-1}\right)=\hat{b}_1z^{-1}+\hat{b}_2z^{-2}+\hat{b}_3z^{-3}$$

$$P\left(z^{-1}\right)=1+p_1z^{-1}+p_2z^{-2} \qquad Q\left(z^{-1}\right)=q_0+q_1z^{-1}+q_2z^{-2}$$

$$R\left(z^{-1}\right)=r_0 \qquad S\left(z^{-1}\right)=s_0+s_1z^{-1}+s_2z^{-2}$$

Source code:

sid on-line identification s-function

db3s function for computation of controller parameters

scrqp controller s-function

See also:
On-line identification methods
List of common controllers parameters

4.33 pp3chp

Generic pole placement controller for processes of third order

Transfer function of the controlled system:

$$G(z)=\frac{b_1z^{-1}+b_2z^{-2}+b_3z^{-3}}{1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}}$$

Vector of identification initial parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{a}_3,\hat{b}_1,\hat{b}_2,\hat{b}_3\right]^{\mathrm{T}}$$

Controller specific parameters:

Parameter	Description
Reference signal type	It's possible to choose one of the following signal types: • steps • ramps • sin waves
Frequency	Frequency *f* of reference signal - valid only when reference signal is 'sin waves'
Coefficients of characteristic polynomial	Vector determining characteristic polynomial $D(z^{-1})$ and thus the positions of the poles of the closed loop. If vector is [*d0, d1, d2, d3, d4. d5*], then characteristic polynomial will be $D(z^{-1}) = d0 + d1\cdot z^{-1} + d2\cdot z^{-2} + d3\cdot z^{-3} + d4\cdot z^{-4} + d5\cdot z^{-5}$. The length of the vector can vary but at lest *d0* must be specified.

Control law:

$$u_k=\frac{1}{p_0}\left(r_0w_k+r_1w_{k-1}-q_0y_k-q_1y_{k-1}-q_2y_{k-2}-p_1u_{k-1}-p_2y_{k-2}\right)$$

where controller parameters r_0, r_1, q_0, q_1, q_2, p_0, p_1 and p_2 are calculated by solving following diophantine equation:

$$A\left(z^{-1}\right)P\left(z^{-1}\right)+B\left(z^{-1}\right)Q\left(z^{-1}\right)=D\left(z^{-1}\right)$$

$$B\left(z^{-1}\right)R\left(z^{-1}\right)+F\left(z^{-1}\right)S\left(z^{-1}\right)=D\left(z^{-1}\right)$$

where polynomials are as follows:

$A\left(z^{-1}\right)=1+\hat{a}_1z^{-1}+\hat{a}_2z^{-2}+\hat{a}_3z^{-3}$

$B\left(z^{-1}\right)=\hat{b}_1z^{-1}+\hat{b}_2z^{-2}+\hat{b}_3z^{-3}$

$P\left(z^{-1}\right)=p_0+p_1z^{-1}+p_2z^{-2}$

$Q\left(z^{-1}\right)=q_0+q_1z^{-1}+q_2z^{-2}$

$R\left(z^{-1}\right)=r_0+r_1z^{-1}$

$S\left(z^{-1}\right)$ is any appropriate polynomial

$F(z^{-1})$ depends on reference signal type:

Reference signal type	$F(z^{-1})$
steps	$1-z^{-1}$
ramps	$1-2z^{-1}+z^{-2}$
sin waves	$1-2\cdot\cos\omega\cdot z^{-1}+z^{-2}$ where $\omega=2\pi f\cdot T_0$

Source code:

sid	on-line identification s-function
pp3chp	function for computation of controller parameters
scast	controller s-function

See also:

On-line identification methods
List of common controllers parameters

4.34 pp3c2dof

Pole placement 2 degree-of-freedom (2DOF) controller with compensator for third order processes.

Transfer function of the controlled system:

$$G(z)=\frac{b_1z^{-1}+b_2z^{-2}+b_3z^{-3}}{1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}}$$

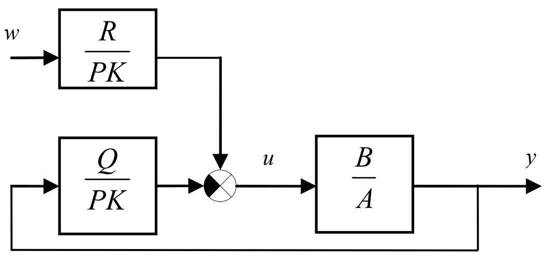
Vector of parameter estimations:

$$\left[\hat{a}_1,\hat{a}_2,\hat{a}_3,\hat{b}_1,\hat{b}_2,\hat{b}_3\right]^{\mathrm{T}}$$

Controller specific parameters:

Parameter	Description
xi – damping factor	Dumping factor ξ specifying dynamic behaviour of closed loop. The dynamic behaviour of the closed-loop is similar to second order continuous system with characteristic polynomial $s^2+2\cdot\xi\cdot\omega\cdot s+\omega^2$.
omega - natural frequency	Natural frequency ω specifying dynamic behaviour of closed loop. The dynamic behaviour of the closed-loop is similar to second order continuous system with characteristic polynomial $s^2+2\cdot\xi\cdot\omega\cdot s+\omega^2$.

2DOF control loop:



Feedback controller:

$$G_R(z)=\frac{Q(z^{-1})}{P(z^{-1})K(z^{-1})}=\frac{q_0+q_1z^{-1}+q_2z^{-2}+q_3z^{-3}}{\left(1+p_1z^{-1}+p_2z^{-2}\right)\left(1-z^{-1}\right)}$$

Feedforward controller for a step reference signal:

$$G_F(z)=\frac{R(z^{-1})}{P(z^{-1})K(z^{-1})}=\frac{r_0}{\left(1+p_1z^{-1}+p_2z^{-2}\right)\left(1-z^{-1}\right)}$$

Characteristic polynomial of closed loop:

$$A(z^{-1})P(z^{-1})K(z^{-1})+B(z^{-1})Q(z^{-1})=D(z^{-1})$$

where polynomials are as follows:

$A(z^{-1})=1+\hat{a}_1z^{-1}+\hat{a}_2z^{-2}+\hat{a}_3z^{-3}$ $B(z^{-1})=\hat{b}_1z^{-1}+\hat{b}_2z^{-2}+\hat{b}_3z^{-3}$

$P(z^{-1})=1+p_1z^{-1}+p_2z^{-2}$ $Q(z^{-1})=q_0+q_1z^{-1}+q_2z^{-2}+q_3z^{-3}$

$$K\left(z^{-1}\right)=1-z^{-1}$$

$$D\left(z^{-1}\right)=1+d_1z^{-1}+\ldots+d_6z^{-6} \qquad d_1=-2\exp\left(-\xi\omega T_0\right)\cos\left(\omega T_0\sqrt{1-\xi^2}\right) \qquad \text{for } \xi\le 1$$

$$d_1=-2\exp\left(-\xi\omega T_0\right)\cosh\left(\omega T_0\sqrt{\xi^2-1}\right) \qquad \text{for } \xi>1$$

$$d_2=\exp\left(-2\xi\,\omega\,T_0\right)$$

$$d_3=d_4=d_5=d_6=0$$

Matrix equation:

3rd order	nth order
$\begin{bmatrix}\hat b_1 & 0 & 0 & 0 & 1 & 0\\ \hat b_2 & \hat b_1 & 0 & 0 & \hat a_1-1 & 1\\ \hat b_3 & \hat b_2 & \hat b_1 & 0 & \hat a_2-\hat a_1 & \hat a_1-1\\ 0 & \hat b_3 & \hat b_2 & \hat b_1 & \hat a_3-\hat a_2 & \hat a_2-\hat a_1\\ 0 & 0 & \hat b_3 & \hat b_2 & -\hat a_3 & \hat a_3-\hat a_2\\ 0 & 0 & 0 & \hat b_3 & 0 & -\hat a_3\end{bmatrix}\begin{bmatrix}q_0\\ q_1\\ q_2\\ q_3\\ p_1\\ p_2\end{bmatrix}=\begin{bmatrix}d_1+1-\hat a_1\\ d_2+\hat a_1-\hat a_2\\ d_3+\hat a_2-\hat a_3\\ d_4+\hat a_3\\ d_5\\ d_6\end{bmatrix}$	$\begin{bmatrix}\hat b_1 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 0\\ \hat b_2 & \hat b_1 & 0 & \cdots & 0 & 0 & 0 & \hat a_1-1 & 1 & 0 & \cdots & 0 & 0\\ \hat b_3 & \hat b_2 & \hat b_1 & \cdots & 0 & 0 & 0 & \hat a_2-\hat a_1 & \hat a_1-1 & 1 & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots\\ \hat b_{n-1} & \hat b_{n-2} & \hat b_{n-3} & \cdots & \hat b_1 & 0 & 0 & \hat a_{n-2}-\hat a_{n-3} & \hat a_{n-3}-\hat a_{n-4} & \hat a_{n-4}-\hat a_{n-5} & \cdots & \hat a_1-1 & 1\\ \hat b_n & \hat b_{n-1} & \hat b_{n-2} & \cdots & \hat b_2 & \hat b_1 & 0 & \hat a_{n-1}-\hat a_{n-2} & \hat a_{n-2}-\hat a_{n-3} & \hat a_{n-3}-\hat a_{n-4} & \cdots & \hat a_2-\hat a_1 & \hat a_1-1\\ 0 & \hat b_n & \hat b_{n-1} & \cdots & \hat b_3 & \hat b_2 & \hat b_1 & \hat a_n-\hat a_{n-1} & \hat a_{n-1}-\hat a_{n-2} & \hat a_{n-2}-\hat a_{n-3} & \cdots & \hat a_3-\hat a_2 & \hat a_2-\hat a_1\\ 0 & 0 & \hat b_n & \cdots & \hat b_4 & \hat b_3 & \hat b_2 & -\hat a_n & \hat a_n-\hat a_{n-1} & \hat a_{n-1}-\hat a_{n-2} & \cdots & \hat a_4-\hat a_3 & \hat a_3-\hat a_2\\ 0 & 0 & 0 & \cdots & \hat b_5 & \hat b_4 & \hat b_3 & 0 & -\hat a_n & \hat a_n-\hat a_{n-1} & \cdots & \hat a_5-\hat a_4 & \hat a_4-\hat a_3\\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & \hat b_n & \hat b_{n-1} & \hat b_{n-2} & 0 & 0 & 0 & \cdots & \hat a_n-\hat a_{n-1} & \hat a_{n-1}-\hat a_{n-2}\\ 0 & 0 & 0 & \cdots & 0 & \hat b_n & \hat b_{n-1} & 0 & 0 & 0 & \cdots & -\hat a_n & \hat a_n-\hat a_{n-1}\\ 0 & 0 & 0 & \cdots & 0 & 0 & \hat b_n & 0 & 0 & 0 & \cdots & 0 & -\hat a_n\end{bmatrix}\begin{bmatrix}q_0\\ q_1\\ q_2\\ \vdots\\ q_{n-2}\\ q_{n-1}\\ q_n\\ p_1\\ p_2\\ p_3\\ \vdots\\ p_{n-3}\\ p_{n-2}\\ p_{n-1}\end{bmatrix}=\begin{bmatrix}d_1+1-a_1\\ d_2+a_1-a_2\\ d_3+a_2-a_3\\ \vdots\\ d_{n-1}+a_{n-2}-a_{n-1}\\ d_n+a_{n-1}-a_n\\ d_{n+1}+a_n\\ d_{n+2}\\ d_{n+3}\\ \\ d_{2n-2}\\ d_{2n-1}\\ d_{2n}\end{bmatrix}$

Control law:

3rd order	nth order
$P(z^{-1})K(z^{-1})u_k=R(z^{-1})w_k-Q(z^{-1})y_k$	
$u_k=r_0w_k-q_0y_k-q_1y_{k-1}-q_2y_{k-2}-q_3y_{k-3}+ +\left(1-p_1\right)u_{k-1}+\left(p_1-p_2\right)u_{k-2}+p_2u_{k-3}$	$u_k=r_0w_k-q_0y_k-q_1y_{k-1}-\ldots-q_ny_{k-n}+ +\left(1-p_1\right)u_{k-1}+\left(p_1-p_2\right)u_{k-2}+\ldots+ +(p_{n-2}-p_{n-1})u_{k-n+1}+p_{n-1}u_{k-n}$
$r_0=\dfrac{1+d_1+d_2+d_3+d_4+d_5+d_6}{b_1+b_2+b_3}$	$r_0=\dfrac{1+d_1+d_2+\ldots+d_{2n}}{b_1+b_2+\ldots+b_n}$

Source code:

sid on-line identification s-function

pp3c2dof function for computation of controller parameters

scfbfw controller s-function

See also:

On-line identification methods
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