VECTOR ALGEBRA TOOLBOX
[MATLAB ® evolves becoming ARRAYLAB ☺]

Vector algebra for arrays of any size, with array expansion enabled

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Summary

Multiple dot, cross, and outer products, cross divisions, Euclidean norms, normalizations, projections, rejections, with automatic virtual array expansion enabled.

Description

This toolbox was written to complete the incomplete set of vectorial operations provided with MATLAB, and to enhance the features of two of them (DOT and CROSS), by enabling virtual array expansion (AX). AX is enabled in all the binary operations included in this toolbox, and allows you, for instance, to multiply a single vector with an array of vectors, by virtually replicating the vector to match the size of the array.

All the functions in this toolbox can automatically operate on *block arrays* (arrays of any size containing vectors along one of their dimensions). MATLAB includes four functions which can apply binary operations on the vectors contained in two block arrays:

- **+** Generic operator usable to perform vector additions.
- **-** Generic operator usable to perform vector subtractions.
- **DOT** Specific function performing dot (inner) products.
- **CROSS** Specific function performing cross products.

They all perform multiple operations, but they cannot perform automatic AX**.** MATLAB also includes function **SUM**, which can be used to add together two or more vectors contained in a single matrix or N-D array. For instance, if A is a M×N matrix, regarded as a concatenation of N column vectors, these vectors can be added together by using SUM(A, 2), which yields an M×1 matrix.

DOT2 and CROSS2, included in this toolbox, enable AX for dot and cross products.

Outer products, orthogonal cross divisions, and generalized cross divisions with AX enabled, can be performed by calling functions OUTER, OCD, and CROSSDIV, included in this toolbox.

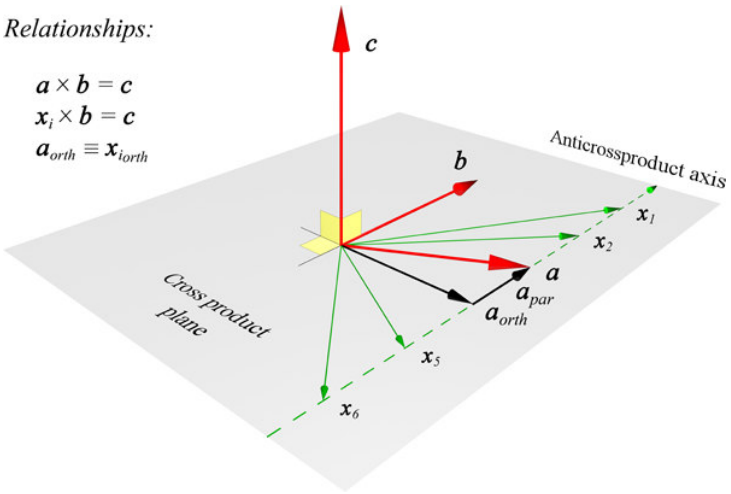

Figure 1. The infinitely many anticrossproducts of the cross product $\boldsymbol{a} \times \boldsymbol{b}$. They are defined as the vectors $\boldsymbol{x}_i$ such that $\boldsymbol{x}_i \times \boldsymbol{b} = \boldsymbol{a} \times \boldsymbol{b}$ (de Leva, 2008).

As for cross divisions (CDs), they were not invented until now (see Weisstein, 2008), but I needed them, so I defined and implemented them (de Leva, 2008). They determine one of the infinitely many anticrossproducts of the cross product $\boldsymbol{a} \times \boldsymbol{b}$ (fig. 1). They were not indispensable, but markedly simplified my equations and my code, and those who appreciate short symbolic equations are likely to love them. Without CDs, in some cases you are forced to write scalar equations, containing negative terms and operations involving various combinations of scalar components. In other cases, you need to write longer vectorial equations. Those equations are not complex, but they are certainly less simple than a CD. For instance, a typo is more likely to occur when you write scalar equations than when you write a CD. See CROSSDIV help for more detailed explanations.

A list of simple vectorial operations is given below. Only the first five are implemented in intrinsic MATLAB functions. The others are implemented in this toolbox. All of them can operate on N-D arrays of vectors. See the respective help texts for further details.

	Operation	Implementation	Additional features
MATLAB ®	Repeated Addition	**SUM**	
	Binary Addition	**+**	
	Subtraction	**-**	
	Dot product	**DOT**	
	Cross product	**CROSS**	
ARRAYLAB ©	Euclidean norm (magnitude)	**MAGN**	
	Normalization	**UNIT**	
	Dot product	**DOT2**	AX enabled
	Cross product	**CROSS2**	AX enabled
	Outer product	**OUTER**	AX enabled
	Orthogonal cross division	**OCD**	AX enabled
	Generalized cross division	**CROSSDIV**	AX enabled
	Projection	**PROJECTION**	AX enabled
	Rejection	**REJECTION**	AX enabled

A detailed definition of AX is given in the manual of the MULTIPROD toolbox, published on MATLAB Central (file #8773).

The ARRAYLAB toolbox

The functions included in this toolbox comply with the ARRAYLAB philosophy, and are a part of the "ARRAYLAB toolbox" (see the manual of the MULTIPROD toolbox, MATLAB Central, file #8773).

Some of them call MULTIPROD. This function is a powerful generalization for N-D arrays of the MATLAB function MTIMES and the matrix multiplication operator (*).

MULTIPROD has a broad field of potential applications. For instance, it can use large arrays of 3×3 or 4×4 transformation matrices to perform, in a single step and with no loops, multiple geometrical transformations (rotations, roto-translations) on arrays of vectors. Thus, it deserves a separate introduction and I published it in a separate package on MATLAB Central (file #8773).

Some other functions of this toolbox call BAXFUN, another function which belongs to the ARRAYLAB toolbox and which is published separately on MATLAB Central (file #23084).

Output testing

The functions testDOT2, testCROSS2, testOUTER, testPROJECTION, testREJECTION, testUNIT, testXDIV, testOCD, stored in a separate folder "Testing", contain the code I used to test the output of the main functions. Since UNIT calls MAGN, testUNIT tests both UNIT and MAGN.

Requirements

Since they represent the engines of this toolbox, MULTIPROD and BAXFUN (MATLAB Central files #8773 and #23084) must be installed in your system.

The users of MATLAB releases prior to R2007a must install Douglas Schwarz's substitute for **bsxfun**, a function which is builtin in MATLAB R2007a and later releases, and which is called by MULTIPROD and BAXFUN.

REFERENCES

de Leva P., 2008. Anticrossproducts and cross divisions. Journal of Biomechanics, 8, 1790-1800 (http://dx.doi.org/doi:10.1016/j.jbiomech.2007.09.030)

Weisstein E. W. Vector Division. From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/VectorDivision.html (Retreived on 2008 Dec 4)