



Improving Mass Moment of Inertia Measurements

By Matt Jardin

Mass moment of inertia is an important parameter for the accurate dynamic modeling of aerospace vehicles and other mechanical systems. For simple bodies, mass moment of inertia can be obtained from a CAD model or derived analytically. For more complex bodies, it must be measured. The bifilar (two support wire) vertical-axis torsional pendulum is a simple and cost-effective device for this purpose (Figure 1).

Products Used

- Simulink®
- Simulink Design Optimization™

Typically, linearized models of the bifilar pendulum are used to estimate mass moment of inertia from the average rotational oscillation period. These linearized models are simpler to use than full nonlinear models, but the simplifications may compromise model accuracy. This article shows how you can improve mass moments of inertia estimates by solving a more accurate nonlinear model of the pendulum.

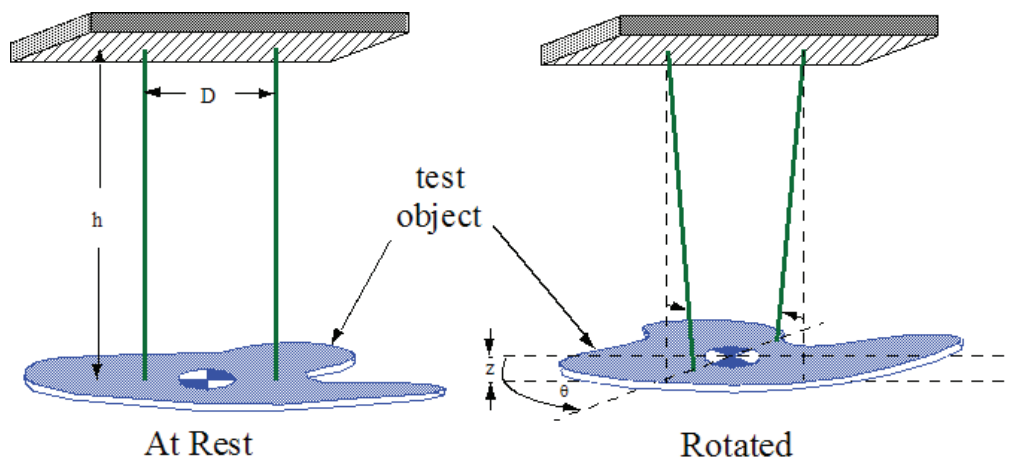


Figure 1. The bifilar torsional pendulum. A test object is suspended from two parallel vertical wires that are free to rotate about their attachment points; the restoring torque of the bifilar pendulum is provided by the gravitational force as rotations from rest cause the test object to rise slightly.

Deriving Mass Moment of Inertia Using a Simplified Linear Equation

Because the bifilar torsional pendulum system is easily defined by its kinetic and potential energy, it lends itself to a Lagrangian dynamics formulation. Omitting the small amount of kinetic energy in the suspension wires and in the raising and lowering of the test object, the total kinetic energy, T , of the pendulum system is the rotational kinetic energy of the test object

$$T = \frac{1}{2} I \dot{\theta}^2$$

where I is the mass moment of inertia about the vertical (z) axis and $\dot{\theta}$ is the rate of change of rotational displacement about the vertical axis. Omitting the energy of the support wires, the gravitational potential energy, V , of the system is given by

$$V = mgz$$

where m is the mass of the test object (and any tare platform being used), g is the gravitational acceleration, and z is the vertical displacement of center of gravity.

The damping associated with rotational motion is modeled by both aerodynamic drag and viscous damping. The generalized damping force model is

$$Q = -K_D \dot{\theta} |\dot{\theta}| - C \dot{\theta}$$

where K_D and C are empirically-determined aerodynamic and viscous damping parameters, respectively.

Using the Lagrangian dynamics formulation, the following equation of motion for the pendulum can be derived¹:

$$\ddot{\theta} + \left(\frac{K_D}{I} \dot{\theta} |\dot{\theta}| + \frac{C}{I} \dot{\theta} \right) + \left(\frac{mgD^2}{4Ih} \right) \frac{\sin \theta}{\sqrt{1 - 0.5(D/h)^2(1 - \cos \theta)}} = 0 \quad (1)$$

While several phenomena have been omitted, this equation captures the key dynamics of the bifilar pendulum.

This nonlinear equation is often simplified by assuming that the angular motion is small, and by omitting damping. This simplification produces a linear equation that yields the expression that can be used to estimate mass moment of inertia:

$$I = \frac{mgD^2}{4h\omega_n^2} \quad (2)$$

This method has the benefit of requiring simple measurements (measuring the frequency of oscillations, ω_n), but the simplicity gained by linearizing and neglecting damping terms comes at the expense of measurement accuracy. Simulink Design Optimization™ enables us to estimate mass moment of inertia using a more precise equation.

¹ Jardin, M. R., Mueller, E. R., "Optimized Measurements of Unmanned-Air-Vehicle Mass Moment of Inertia with a Bifilar Pendulum." *Journal of Aircraft*, Vol. 46, No. 3, May-June 2009, pp. 163-775.

Estimating Mass Moment of Inertia Using the Nonlinear Equation

Our goal is to determine which value of the mass moment of inertia causes our simulated pendulum motion to best fit measured data on the pendulum's motion. We begin by creating a Simulink® model of the bifilar pendulum that implements equation 1 (Figure 2). The model accepts the parameters-of-interest as inputs (Figure 3).

We create the model by designing a library block subsystem for the bifilar pendulum with mask parameters for defining pendulum dimensions and estimated error standard deviations (Figure 3). The error standard deviation parameters let us explore the effects of pendulum measurement errors on the resulting estimates of moment of inertia. We use inputs to the bifilar pendulum library block to define test object parameters. We can then use Simulink Design Optimization to automatically vary the test object parameters so that the simulation data matches recorded experimental data.

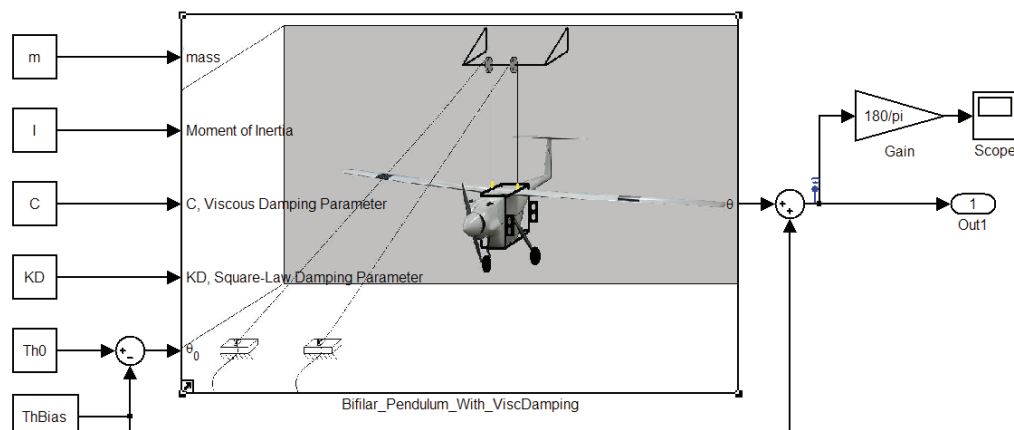


Figure 2. Simulink model of the bifilar pendulum.

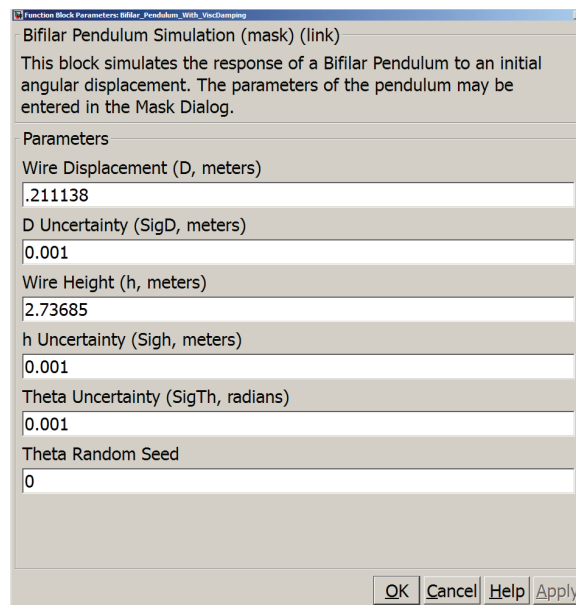


Figure 3. Mask dialog for the Bifilar Pendulum block.

We set up the parameter estimation problem in the Control and Estimation Tools Manager by completing the following steps:

- Import measurement data and associate that data with an output port of the model.
- Define and configure the parameters that we want to estimate.
- Configure the estimation options.
- Run the estimation task.

Importing Measurement Data

We begin by loading experimental data into the workspace and importing it using the Control and Estimation Tools Manager (Figure 4). This data represents the rotation angle vs. time for an actual pendulum test object.

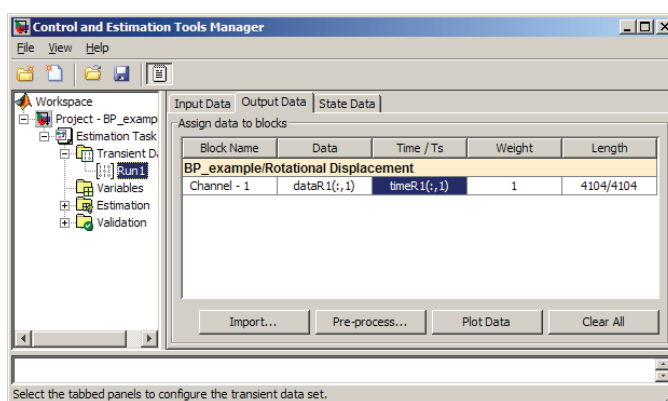


Figure 4. Control and Estimation Tools Manager Output Data tab.

Clicking the “Plot Data” button displays a basic plot of the data we have imported (Figure 5).

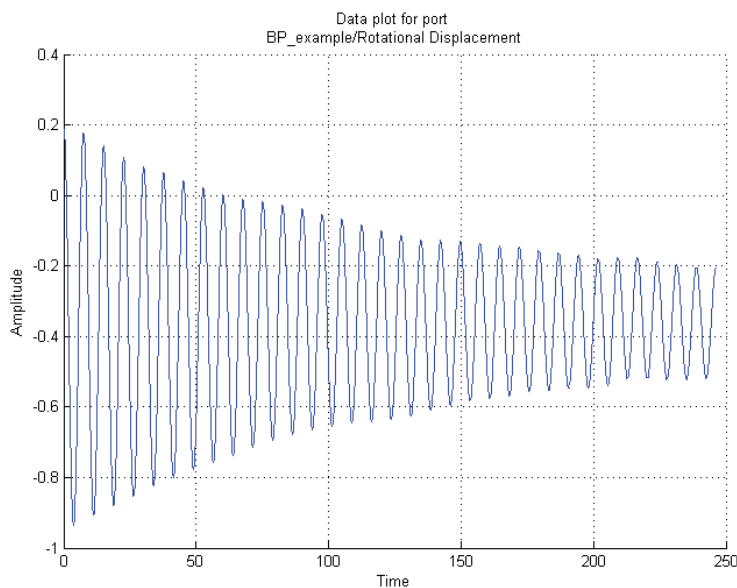


Figure 5. Transient test data.

Selecting and Configuring Estimation Parameters

The Estimated Parameters dialog in Simulink Design Optimization lets us select parameters from the Base Workspace to be varied (Figure 6).

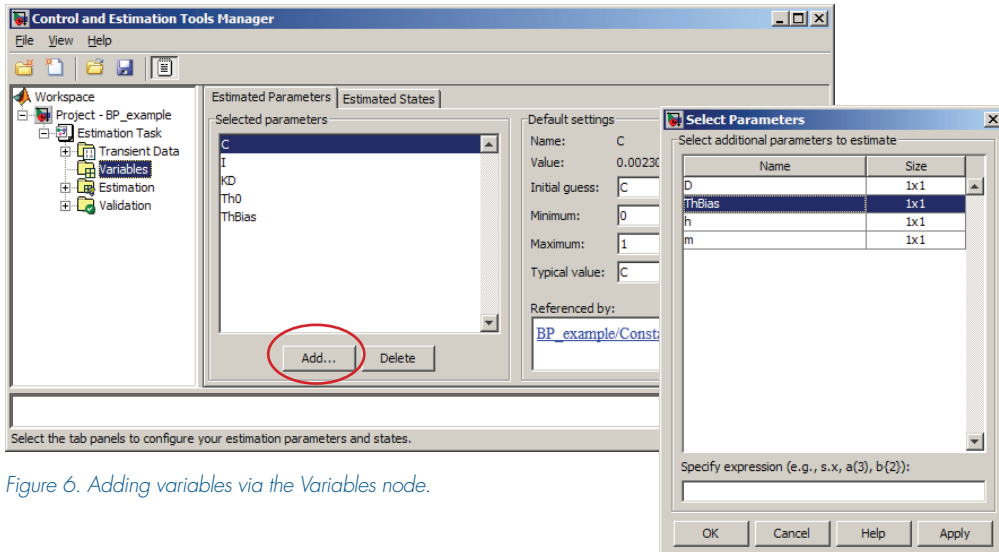


Figure 6. Adding variables via the Variables node.

The right-hand pane of the Variables node lets us set properties of the selected parameters, such as the initial guess, minimum and maximum allowable value, and typical value. These properties help guide and constrain the parameter estimation process.

Configuring Estimation Options

The Estimation node enables us to set options to modify the estimation process (Figure 7).

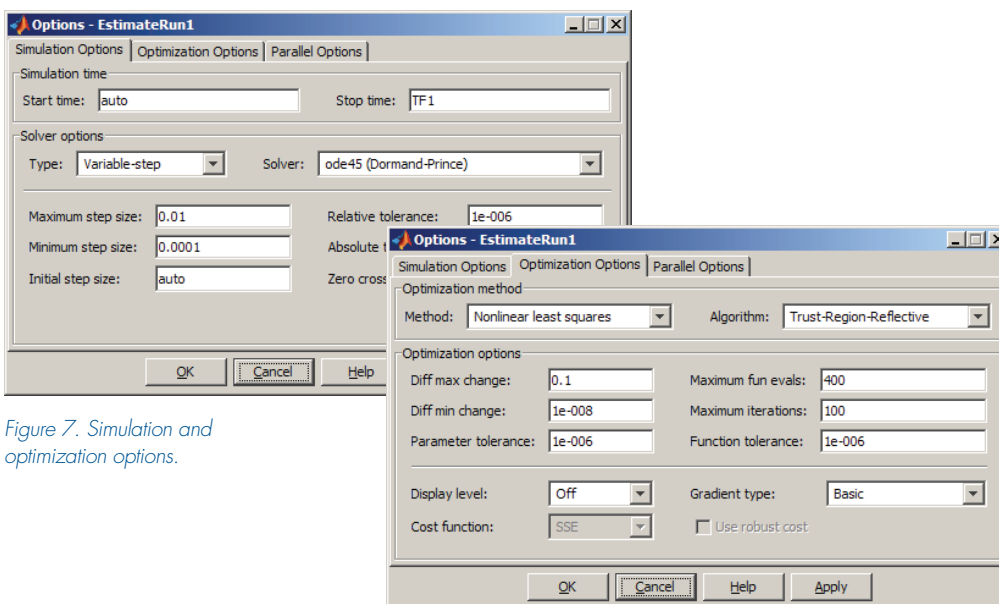


Figure 7. Simulation and optimization options.

Using options on these dialogs we can change the way Simulink runs the simulation and performs the parameter estimation.

Running the Estimation

With the estimation problem properly configured, we run the estimation by clicking “Start” on the Estimation tab of the Estimation node. The estimation progress is displayed in the middle pane (Figure 8).

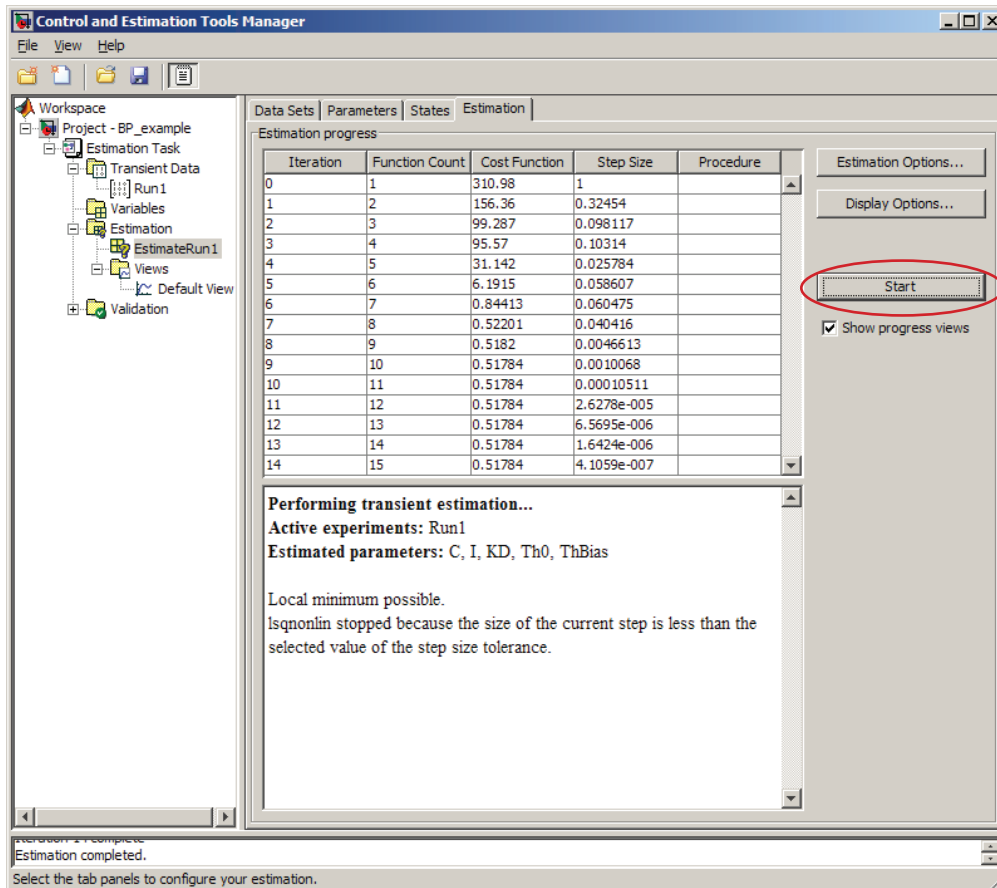


Figure 8. Running the estimation.

Once the step size is within the specified optimization tolerance, the estimation process is completed and we can view the final parameter values. We could also visualize the intermediate results in a series of plots (Figure 9). Simulation and parameter trajectory plots are useful for monitoring the progress of the parameter optimization process.

After three iterations, the plots show that the estimation algorithm matches the oscillation frequency but that the damping level is too high. The estimation completes after 14 iterations in this case, and shows a close match between the simulated and the measured data. This result gives us high confidence that the estimated parameter values are accurate.

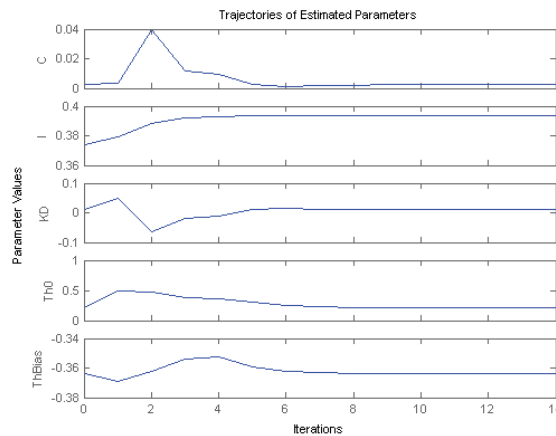
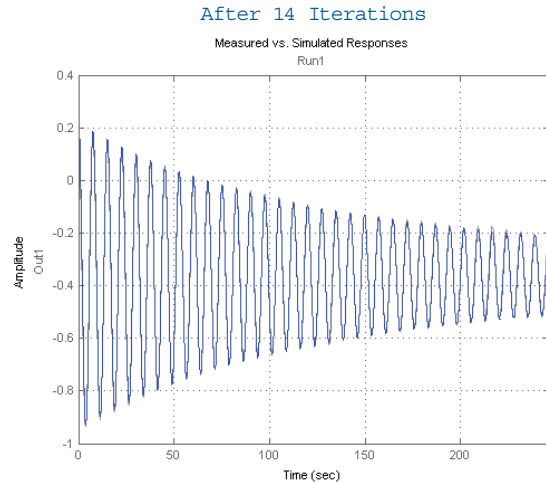
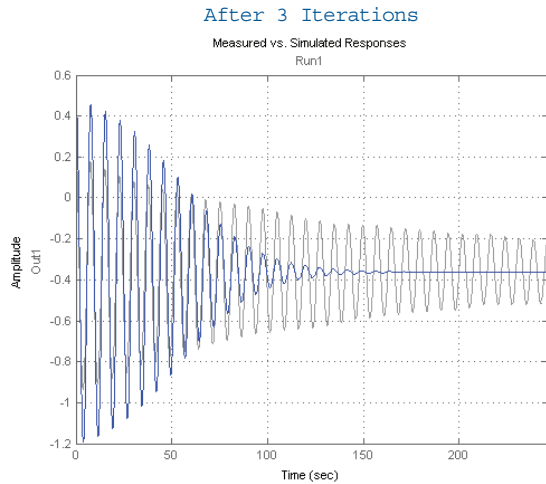


Figure 9. Plots of the estimation progress. Top left: Measured vs. simulated response at the beginning of optimization. Top right: Measured vs. simulated response at the end of optimization. Bottom: Estimated parameter trajectories during optimization.

Extending This Approach

We could further refine this approach by using SimMechanics™ to model the motion of the pendulum. With SimMechanics we would use the physical topography of the system to create a model instead of explicitly deriving the equation. As a result, a complex system like this one could be modeled more quickly and easily, and with higher accuracy. We could then use Simulink Design Optimization to estimate model parameters to obtain even more accurate measurement of mass moment of inertia. ■

For More Information

- **Webinar:** *Introduction to Simulink Design Optimization*
www.mathworks.com/wbnr33133
- **Demo:** *Inverted Pendulum Parameter Estimation*
www.mathworks.com/inverted-pendulum

For Download

- **Simulink®** model of a bifilar pendulum for measuring mass moment of inertia
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