MATLAB COMPUTATIONAL FINANCE CONFERENCE
New York City, April 9 2014

Attilio Meucci

(Re)Defining and Managing Diversification

STUDY IT:  www.symmys.com (white papers and code)

DO IT:  Advanced Risk and Portfolio Management® Bootcamp
  www.symmys.com/arpm-bootcamp
Standard approach: Modern Portfolio Theory

\[ w^* \equiv \arg \max_{w \in C} (E\{R\} - \lambda \sqrt{V\{R\}}), \]

Optimal Portfolio = optimal mean-variance weights under constraints
Standard approach: Modern Portfolio Theory

\[ R = \sum_{n=1}^{n} w_n R_n. \]

Portfolio Return = weighted average of asset returns

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Standard approach: Modern Portfolio Theory

Portfolio Return = weighted average of asset returns

\[ R = \sum_{n=1}^{\bar{n}} w_n R_n. \]

Optimal Portfolio = optimal mean-variance weights under constraints

\[ w^* = \arg\max_{w \in C} (\mathbb{E}\{R\} - \lambda \sqrt{\mathbb{V}\{R\}}). \]

\[ \mathbb{E}\{R\} = w' \mu_R \]
\[ \mathbb{V}\{R\} = w' \Sigma_R w \]

\( \bar{n} \times 1 \) vector of expected asset returns

\( \bar{n} \times \bar{n} \) matrix of asset covariances
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Trend 1: From asset-based allocation to factor-based allocation

\[ \max (E\{ R \} - \lambda \sqrt{V\{ R \}}). \]

Optimal Portfolio = optimal mean-variance
Trend 1: From asset-based allocation to factor-based allocation

\[ R = \sum_{k=1}^{\hat{k}} b_k F_k + U, \]
\[ = \sum_{k=0}^{\hat{k}} b_k F_k, \]
\[ = \sum_k b_k F_k, \]

Portfolio Return = Linear factor model
(factors: momentum, value, ..., PCA, ...
strategies, ...)

Optimal Portfolio = optimal mean-variance

\[ \max \{ \mathbb{E} \{ R \} - \lambda \mathbb{V} \{ R \} \}. \]
Trend 1: From asset-based allocation to factor-based allocation

\[ R = \sum_{k=1}^{\hat{k}} b_k F_k + U, \]
\[ = \sum_{k=0}^{\hat{k}} b_k F_k, \]
\[ = \sum_{k} b_k F_k, \]

Portfolio Return = Linear factor model
(factors: momentum, value, ..., PCA, ...
strategies,...)

Optimal Portfolio = optimal mean-variance exposures under constraints
Trend 1: From asset-based allocation to factor-based allocation

\[ R = \sum_{k=1}^{\tilde{k}} b_k F_k + U, \]
\[ = \sum_{k=0}^{\tilde{k}} b_k F_k, \]
\[ = \sum_{k} b_k F_k, \]

Portfolio Return = Linear factor model
(factors: momentum, value, ..., PCA, ...
strategies, ...)

Optimal Portfolio = optimal mean-variance
exposures under constraints

\[ b^* = \arg\max_{b \in \mathcal{C}} \left( \mathbb{E}\{R\} - \lambda \mathbb{V}\{R\} \right). \]

\[ \mathbb{E}\{R\} = b' \mu_F \]
\[ \mathbb{V}\{R\} = b' \Sigma_F b \]

\[ \tilde{k} \times 1 \text{ vector of factor premia} \]
\[ \tilde{k} \times \tilde{k} \text{ matrix of factor covariances} \]
Trend 2: from mean-variance to risk parity, or diversification management

\[ R = \sum_{n=1}^{\tilde{n}} w_n R_n. \]

Portfolio Return = weighted average of asset returns

\[ m_n \equiv \frac{1}{Sd\{R\}} \frac{\partial Sd\{R\}}{\partial w_n} = \frac{w_n [\Sigma_R w]_n}{w' \Sigma_R w}, \]

“contributions” to risk
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**Diversification:** risk “contributions”

Trend 2: from mean-variance to risk parity, or diversification management

\[ R = \sum_{n=1}^{\tilde{n}} w_n R_n. \]

Portfolio Return = weighted average of asset returns

\[
m_1 = m_2 = \cdots
\]

Optimal Portfolio = equal “contributions” to risk

\[
m_n \equiv \frac{1}{\text{sd}\{R\}} \frac{\partial \text{sd}\{R\}}{\partial w_n} = \frac{w_n [\Sigma_R w]_n}{w' \Sigma_R w},
\]

“contributions” to risk

\[ \tilde{n} \times \tilde{n} \text{ matrix of asset covariances} \]
Portfolio Return = \( \sum_{n=1}^{n} w_n R_n \).

Portfolio Return = weighted average of asset returns.
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Diversification: Effective Number of Bets

\[ R_w \equiv w' R \]

\[ \text{Var} \{ R_w \} \equiv \sum_{n=1}^{N} \text{Var} \{ w_n R_n \} \]

if correlations = 0
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Diversification: Effective Number of Bets

\[ R_w \equiv w' R. \]

\[ \text{Var} \{ R_w \} \neq \sum_{n=1}^{N} \text{Var} \{ w_n R_n \} \]

if correlations \( \neq 0 \)
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Diversification: Effective Number of Bets

\[ R_w \equiv w'R. \]

\[ \Sigma \equiv \text{Cov}\{R\} \]

\[ \Sigma \equiv \mathbf{E} \Lambda \mathbf{E}' \]

\[ \mathbf{E} \equiv (e_1, \ldots, e_N) \quad \text{eigenvectors} \]

\[ \Lambda \equiv \text{diag}\left(\lambda_1^2, \ldots, \lambda_N^2\right) \quad \text{eigenvalues} \]
Diversification: Effective Number of Bets

\[ R_w \equiv w'R. \]

\[ \Sigma \equiv \text{Cov}\{R\} \]

\[ '\Sigma \equiv E \Lambda E' \quad \text{PCA} \]

\[ E \equiv (e_1, \ldots, e_N) \quad \text{eigenvectors} \]

\[ e_n \equiv \arg\max_{e'e \equiv 1} \{e'\Sigma e\} \quad \text{uncorrelated, maximum variance portfolios} \]

\[ e'\Sigma e_j \equiv 0, \text{ for all existing } e_j \]

\[ \Lambda \equiv \text{diag}(\lambda_1^2, \ldots, \lambda_N^2) \quad \text{eigenvalues} \]
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Diversification: Effective Number of Bets

\[ R_w \equiv w' \Sigma \]

\[ \Sigma \equiv \text{Cov}\{R\} \]

\[ '\Sigma \equiv E \Lambda E' \]

**PCA**

\[ E \equiv (e_1, \ldots, e_N) \]

**eigenvectors**

\[ e_n \equiv \text{argmax} \{e'\Sigma e\} \quad \text{for } e'e=1 \]

**uncorrelated, maximum variance portfolios**

\[ e'\Sigma e_j \equiv 0, \text{ for all existing } e_j \]

\[ \Lambda \equiv \text{diag} (\lambda_1^2, \ldots, \lambda_N^2) \]

**eigenvalues**

\[ \lambda_n^2 \equiv \text{Var} \{e'_n R\} \]

**variances of uncorrelated, maximum variance portfolios**
Diversification: Effective Number of Bets

\[ R_w \equiv w' \Sigma \]

\[ \Sigma \equiv \text{Cov}\{\mathbf{R}\} \]

\[ '\Sigma \equiv \mathbf{E} \Lambda \mathbf{E}' \]

**PCA**

\[ \mathbf{E} \equiv (e_1, \ldots, e_N) \]

\[ e_n \equiv \arg\max_{e'\in\mathbb{R}^N} \{e'\Sigma e\} \]

Principal portfolios

\[ e'\Sigma e_j = 0, \text{ for all existing } e_j \]

**Eigenvalues**

\[ \Lambda \equiv \text{diag}(\lambda_1^2, \ldots, \lambda_N^2) \]

\[ \lambda_n^2 \equiv \text{Var}\{e_n'\mathbf{R}\} \]

**Principal variances**

**Eigenvectors**
\[ R_w \equiv w' R. \]

\[ \Sigma \equiv \text{Cov}\{R\} \]

\[ '\Sigma \equiv E \Lambda E' \]

\[ E \equiv (e_1, \ldots, e_N) \]

\[ \Lambda \equiv \text{diag}(\lambda_1^2, \ldots, \lambda_N^2) \]
\[ R_w \equiv w' \Sigma w \]

\[ \Sigma \equiv \text{Cov}\{R\} \]

\[ '\Sigma \equiv \Sigma \Lambda \Sigma' \]

\[ \tilde{R} \equiv \Sigma^{-1} R \text{ \quad return of principal portfolios} \]
\[ R_w \equiv w' \mathbf{R} \]

\[ \Sigma \equiv \text{Cov}\{\mathbf{R}\} \]

\[ \Sigma \equiv \mathbf{E} \Lambda \mathbf{E}' \]

\[ \tilde{\mathbf{R}} \equiv \mathbf{E}^{-1} \mathbf{R} \]

return of principal portfolios

\[ \tilde{\mathbf{w}} \equiv \mathbf{E}^{-1} \mathbf{w} \]

weights of original portfolio on principal portfolios
Diversification: Effective Number of Bets

\[
R_w \equiv w' R.
\]

\[
\Sigma \equiv \text{Cov}\{R\}
\]

\[
\Sigma \equiv E \Lambda E'
\]

\[
\tilde{R} \equiv E^{-1} R \quad \text{return of principal portfolios}
\]

\[
\tilde{w} \equiv E^{-1} w \quad \text{weights of original portfolio on principal portfolios}
\]

\[
R_w \equiv \tilde{w}' \tilde{R}.
\]
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Diversification: Effective Number of Bets

\[ R_w \equiv w'R. \]

\[ \text{Var} \{ R_w \} \neq \sum_{n=1}^{N} \text{Var} \{ w_n R_n \} \]

Variance concentration curve

\[ v_n \equiv \tilde{w}_n^2 \lambda_n^2 \]

Principal portfolio number

\[ \tilde{R} \equiv E^{-1}R \]

Return of principal portfolios

\[ \tilde{w} \equiv E^{-1}w. \]

Weights of original portfolio on principal portfolios

\[ v_n \equiv \tilde{w}_n^2 \lambda_n^2 \]

Variance concentration curve

Contribution to original portfolio variance from n-th principal portfolio:

\[ \text{Var} \{ R_w \} \equiv \sum_{n=1}^{N} v_n \]
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Diversification: Effective Number of Bets

\[ R_w \equiv \tilde{w}'\tilde{R}. \]

volatility concentration curve

\[ \tilde{R} \equiv E^{-1}R \]

return of principal portfolios

\[ \tilde{w} \equiv E^{-1}w. \]

weights of original portfolio on principal portfolios

\[ \nu_n \equiv \tilde{\omega}_n^2 \lambda_n^2. \]

variance concentration curve

\[ s_n \equiv \frac{\tilde{\omega}_n^2 \lambda_n^2}{\text{Sd} \{R_w\}}. \]

volatility concentration curve

contribution to original portfolio volatility from n-th principal portfolio: “hot spots”
$R_w \equiv w' \mathbb{R}.$

Diversification distribution

\[ \begin{align*}
\tilde{R} &\equiv \mathbb{E}^{-1} \mathbb{R} \\
\tilde{w} &\equiv \mathbb{E}^{-1} w,
\end{align*} \]

return of principal portfolios

weights of original portfolio on principal portfolios

\[ \begin{align*}
\nu_n &\equiv \tilde{w}_n^2 \lambda_n^2 \\
\sigma_n &\equiv \frac{\tilde{w}_n^2 \lambda_n^2}{\text{Sd} \{R_w\}} \\
p_n &\equiv \frac{\tilde{w}_n^2 \lambda_n^2}{\text{Var} \{R_w\}}
\end{align*} \]

variance concentration curve

volatility concentration curve

diversification distribution

contribution to original portfolio r-square from n-th principal portfolio
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\[ R_w \equiv w'\mathbf{R}. \]

\[ \mathbf{R} \equiv \mathbf{E}^{-1}\mathbf{R} \]

\[ \mathbf{w} \equiv \mathbf{E}^{-1}\mathbf{w}. \]

\[ \nu_n \equiv \tilde{w}_n^2 \lambda_n^2 \]

\[ s_n \equiv \frac{\tilde{w}_n^2 \lambda_n^2}{\text{Sd}\{R_w\}} \]

\[ p_n \equiv \frac{\tilde{w}_n^2 \lambda_n^2}{\text{Var}\{R_w\}} \]

return of principal portfolios

weights of original portfolio on principal portfolios

variance concentration curve

volatility concentration curve

diversification distribution

Diversification: Effective Number of Bets

\[ R_w \equiv \tilde{w}'\mathbf{R}. \]
weights of original portfolio on principal portfolios

\[ \mathbf{w} = \mathbf{E}^{-1} \mathbf{w}. \]

return of principal portfolios

\[ \mathbf{R} \equiv \mathbf{E}^{-1} \mathbf{R}. \]

variance concentration curve

\[ \nu_n \equiv \tilde{w}_n^2 \lambda_n. \]

\[ s_n \equiv \frac{\tilde{w}_n^2 \lambda_n^2}{\text{Sd} \{ R_w \}}. \]

volatility concentration curve

\[ \nu_n \equiv \tilde{w}_n^2 \lambda_n^2 \]

\[ s_n \equiv \frac{\tilde{w}_n^2 \lambda_n^2}{\text{Var} \{ R_w \}}. \]

diversification distribution: "probability mass"

\[ \rho_n \equiv \frac{\tilde{w}_n^2 \lambda_n^2}{\text{Var} \{ R_w \}}. \]

Diversification: Effective Number of Bets

\[ R_w \equiv \mathbf{\tilde{w}}' \mathbf{\tilde{R}}. \]
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Diversification: Effective Number of Bets

$$N_{Ent} \equiv \exp \left( - \sum_{n=1}^{N} p_n \ln p_n \right)$$

\[ \downarrow \]
diversification

$$\tilde{R} \equiv \text{E}^{-1} R$$
return of principal portfolios

$$\tilde{w} \equiv \text{E}^{-1} w$$
weights of original portfolio on principal portfolios

$$v_n \equiv \tilde{w}_n^2 \lambda_n^2$$
variance concentration curve

$$s_n \equiv \frac{\tilde{w}_n^2 \lambda_n^2}{\text{Sd} \{R_w\}}$$
volatility concentration curve

$$p_n \equiv \frac{\tilde{w}_n^2 \lambda_n^2}{\text{Var} \{R_w\}}$$
diversification distribution: “probability mass”

$$R_w \equiv \tilde{w}' \tilde{R}.$$
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Diversification: Effective Number of Bets

Effective number of bets

$$N_{\text{Ent}} \equiv \exp \left( - \sum_{n=1}^{N} p_n \ln p_n \right)$$

Full concentration

$$N_{\text{Ent}} \approx 1$$

Weights

Diversification distribution: "probability mass"

$$p_n \equiv \frac{\hat{w}_n^2 \lambda_n^2}{\text{Var} \{ R_w \}}$$
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Diversification: Effective Number of Bets

**Effective number of bets**

\[ N_{\text{Ent}} \equiv \exp \left( - \sum_{n=1}^{N} p_n \ln p_n \right) \]

- **Full concentration**
  \[ N_{\text{Ent}} \approx 1 \]
- **Full diversification**
  \[ N_{\text{Ent}} \approx N \]

\[ p_n \equiv \frac{\tilde{w}_n^2 \lambda_n^2}{\text{Var} \{ R_w \}} \]

Diversification distribution: “probability mass”
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**Diversification management**

Effective number of bets

\[
N_{Ent} \equiv \exp \left( - \sum_{n=1}^{N} p_n \ln p_n \right)
\]

Full concentration

\[N_{Ent} \approx 1\]

Full diversification

\[N_{Ent} \approx N\]

Mean-diversification frontier

\[
w_\varphi \equiv \arg\max_{w \in C} \{ \varphi \mu'w + (1 - \varphi) N_{Ent}(w) \}
\]

Weights

Diversification distribution

Current portfolio
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Next Steps: Minimal Torsion Bets

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                             2) Factors on Demand  
                         + Minimal Torsion Bets |
### Factors on Demand

**Premia**
- Modern Portf. Theory

**Parity**
- 1) Marg. Contribs
- 2) Effective Num. Bets

**Factors**
- Factor-Based Allocation
- 1) Marg. Contribs
- 2) Factors on Demand + Minimal Torsion Bets

i) Factors on Demand: \( R = \sum_{k=1}^{K} b_k F_k + U \), best portfolio-specific linear factor model
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Next Steps: Minimal Torsion Bets

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i) Factors on Demand \( R = \sum_{k=1}^{k} b_k F_k + U \), **best** portfolio-specific linear factor model

ii) Minimal Torsion Bets: \( \hat{F}_{MT} \equiv \hat{t}_{MT} F \) uncorrelated factors closest to factors

\[
\text{argmin}_{\text{Cr}\{tF\}=i\tilde{n}\times\tilde{n}} \sqrt{\sum_{n} \forall \{Z_n - (F_n/Sd\{F_n\})\}}
\]
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Next Steps: Minimal Torsion Bets

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i) Factors on Demand $R = \sum_{k=1}^{k} b_k F_k + U$, best portfolio-specific linear factor model

ii) Minimal Torsion Bets: $\hat{F}_{MT} \equiv \hat{t}_{MT} F$ uncorrelated factors closest to factors

$$\arg\min_{\text{Cr}\{tF\} = i \times \bar{n}} NTE\{tF, F\} \approx dg(\sigma_F) c^{-1} dg(\sigma_F)^{-1}$$

$\sqrt{\sum_{n} \mathbb{V} \{Z_n - (F_n/Sd\{F_n\})\}}$  

Riccati root of correlation
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Next Steps: Minimal Torsion Bets

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i) Factors on Demand

\[ R = \sum_{k=1}^{K} b_k F_k + U, \quad \text{best} \quad \text{portfolio-specific linear factor model} \]

ii) Minimal Torsion Bets:

\[ \hat{F}_{MT} \neq \tilde{t}_{MT} F \quad \text{uncorrelated factors closest to factors} \]

\[
\arg \min_{Cr\{tF\} = \bar{n} \times \bar{n}} \ NTE\{tF, F\} \approx dg(\sigma_F) c^{-1} dg(\sigma_F)^{-1} \]

Riccati root of correlation

ii) Diversification Distribution

\[ p_{MT}(b) = \left( \hat{t}^{-1}_{MT} b \right) \circ \left( \hat{t}_{MT} \Sigma_F b \right) \]

\[ \Rightarrow \quad \mathbb{N}_{MT}(b) = e^{-p_{MT}(b)} \ln p_{MT}(b) \]

and

Effective Number of Minimum Torsion Bets
Effective Number of Bets
(normalized)

Minimum-Torsion Diversification Distribution

Portfolio Weights

Weights / Probabilities

Stocks

Time

Sep-12, May-10, Jan-08, Sep-05, May-03, Jan-01
The Effective Number of PCA Bets in the S&P500 is close to 1, since the first PCA factor loadings are similar to the weights of the stocks in the S&P500.

The Effective Number of Minimal Torsion Bets in the S&P500 yields intuitive results.

Normalized Effective Number of Bets = \( \frac{ENB}{N} \)
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Marginal Contributions

Portfolio Weights

Weights / Probabilities

Stocks

Time
### Next Steps: Minimal Torsion Bets

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<th>Diversification Distributions</th>
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<td>Expression</td>
<td>( m = \frac{b\circ(\Sigma_F b)}{b'\Sigma_F b} )</td>
<td>( p = \frac{(t_{MT}^{-1} b)\circ(t_{MT} \Sigma_F b)}{b'\Sigma_F b} )</td>
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<td>Meaning</td>
<td>spurious contributions from original factors</td>
<td>proper contributions from Minimum-Torsion Bets</td>
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<tr>
<td>Properties</td>
<td>( \sum_n m_n = 1, \quad m_n \leq 0 )</td>
<td>( \sum_k p_n = 1, \quad p_n \geq 0 )</td>
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Effective Number of Bets:  http://symmys.com/node/199

Factors on Demand:  http://symmys.com/node/164

Minimal Torsion Bets:  http://symmys.com/node/599