Challenges in xVA Pricing and Risk

Andrew McClelland, PhD
Director of Quantitative Research
Numerix

Matlab Computational Finance Conference
NYC 2017

September 28, 2017
Presentation Outline

- Quick overview of Numerix and why we prioritize xVA
- What is xVA? Some intuition for how things come together
- Computational difficulties encountered in xVA
- Multi-factor hybrid models and American Monte Carlo
- Matlab for complicated exposures modeling and aggregations

---

1 Used when referring to Least-Squares Monte Carlo, a la Longstaff and Schwartz ('01), in the context of exposures calculations.
Numerix\textsuperscript{2} founded in ’96, headquartered in NYC, 250+ employees

- It has origins as a pricing & risk library for derivatives
- Oriented towards exotic & multi-factor derivatives, eg. PRDCs
- Managing risk for exotics is a very complicated process
- Requires sophisticated & delicate models, methods, calibrations etc.

\textsuperscript{2}https://www.numerix.com.
Numerix founded in '96, headquartered in NYC, 250+ employees

It has origins as a pricing & risk library for derivatives

Oriented towards exotic & multi-factor derivatives, eg. PRDCs

Managing risk for exotics is a very complicated process

Requires sophisticated & delicate models, methods, calibrations etc.

Figure: Standard pricing workflow. First, market data such as volatility surfaces are sourced, transformed and cleansed. Second, a pricing model is calibrated to the market data. Third, a pricing method such as Monte Carlo is invoked to compute the price.
Figure: Standard risk workflow. Risk factors among market data and model elements, eg. yield nodes and model parameters, are identified as risk factors. Sensitivities against these factors are computed and are hedged using vanilla instruments.
**Figure:** Standard risk workflow. Risk factors among market data and model elements, e.g., yield nodes and model parameters, are identified as risk factors. Sensitivities against these factors are computed and are hedged using vanilla instruments.
**Figure:** Standard risk workflow. Risk factors among market data and model elements, e.g. yield nodes and model parameters, are identified as risk factors. Sensitivities against these factors are computed and are hedged using vanilla instruments.
After crisis exotics volumes dropped but xVAs, eg. CVA, emerged

- Big challenge for banks and managing xVA resembles managing exotics
- Nice use case for how our clients use a mix of NX and Matlab
- Content should be interesting for this audience also
- Sits within larger issue of holistic modeling of trade impact
After crisis exotics volumes dropped but xVAs, eg. CVA, emerged

Big challenge for banks and managing xVA resembles managing exotics

Nice use case for how our clients use a mix of NX and Matlab

Content should be interesting for this audience also

Sits within larger issue of *holistic* modeling of trade impact
Numerix Matlab Interface

Figure: Example of a CVA calculation using the Matlab interface for NX in combination with native Matlab functionality.
After crisis exotics volumes dropped but xVAs, eg. CVA, emerged

Big challenge for banks and managing xVA resembles managing exotics

Nice use case for how our clients use a mix of NX and Matlab

Content should be interesting for this audience also

Sits within larger issue of holistic modeling of trade impact
After crisis exotics volumes dropped but xVAs, eg. CVA, emerged

Big challenge for banks and managing xVA resembles managing exotics

Nice use case for how our clients use a mix of NX and Matlab

Content should be interesting for this audience also

Sits within larger issue of holistic modeling of trade impact
What is xVA?

- Traditional valuations involve the risk-neutral valuation formula

### Classical Valuation

$$V(t) = \tilde{E}_t \left[ e^{-\int_t^T r(u) du} C(T) \right]$$

- Many ways to derive this formula, equilibrium⁴, arbitrage⁵, etc.
- Hedging approach⁶ is a bit restrictive but mimics bank practice
- Classical hedging assumptions ignore certain “frictions”
- *eg.* counterparty credit risk, funding spreads & collateral

---

⁴ *eg.* Cox, Ingersol and Ross ('85).
⁵ *eg.* Harrison and Kreps ('79).
⁶ *eg.* Black and Scholes ('73) or Burgard and Kjaer ('11).
What is xVA?

- Traditional valuations involve the risk-neutral valuation formula

\[
V(t) = \tilde{\mathbb{E}}_t \left[ e^{-\int_t^T r(u) du} C(T) \right]
\]

- Many ways to derive this formula, equilibrium\(^7\), arbitrage\(^8\), etc.

- Hedging approach\(^9\) is a bit restrictive but mimics bank practice

- Classical hedging assumptions ignore certain “frictions”

- eg. counterparty credit risk, funding spreads & collateral

\(^7\) eg. Cox, Ingersol and Ross (’85).
\(^8\) eg. Harrison and Kreps (’79).
\(^9\) eg. Black and Scholes (’73) or Burgard and Kjaer (’11).
Valuations via Hedging Arguments

Figure: Basic considerations within hedging derivation of valuation rule.
What is xVA?

- Traditional valuations involve the risk-neutral valuation formula

\[
V(t) = \mathbb{E}_t \left[ e^{-\int_t^T r(u)\,du} C(T) \right]
\]

- Many ways to derive this formula, equilibrium\(^{10}\), arbitrage\(^{11}\), etc.

- Hedging approach\(^{12}\) is a bit restrictive but mimics bank practice

- Classical hedging assumptions ignore certain “frictions”

- *eg.* counterparty credit risk, funding spreads & collateral

---

\(^{10}\) *eg.* Cox, Ingersol and Ross (’85).

\(^{11}\) *eg.* Harrison and Kreps (’79).

\(^{12}\) *eg.* Black and Scholes (’73) or Burgard and Kjaer (’11).
Valuations via Hedging Arguments

Figure: Modern considerations within hedging derivation of valuation rule, including counterparty credit risk, funding costs, initial margin requirements.
Valuations via Hedging Arguments

Figure: Modern considerations within hedging derivation of valuation rule, including counterparty credit risk, funding costs, initial margin requirements.
Figure: Modern considerations within hedging derivation of valuation rule, including counterparty credit risk, funding costs, initial margin requirements.
Figure: Modern considerations within hedging derivation of valuation rule, including counterparty credit risk, funding costs, initial margin requirements.
Figure: Modern considerations within hedging derivation of valuation rule, including counterparty credit risk, funding costs, initial margin requirements.
What is CVA?

- Counterparty credit risk $\implies$ “credit valuation adjustment"\(^{13}\) or CVA
- Basically upfront cost of dynamically hedging counterparty default
- Must be added to classical value, or cooked into deal payments

Credit Valuation Adjustment

$$CVA(t) = \widetilde{E}_t \left[ \text{Lifetime Discounted Counterparty Loss}(t, T) \right]$$

- IFRS13 (accounting standard) requires CVA be recorded in statements
- Basel III extended capital framework to include a CVA charge

\(^{13}\)eg. Duffie and Canabarro (’04) and Gregory (’15).
What is CVA?

- Counterparty credit risk $\implies$ “credit valuation adjustment”\(^{14}\) or CVA
- Basically upfront cost of dynamically hedging counterparty default
- Must be added to classical value, or cooked into deal payments

**Credit Valuation Adjustment**

\[
CVA(t) = \tilde{E}_t \left[ \text{Lifetime Discounted Counterparty Loss}(t, T) \right]
\]

- IFRS13 (accounting standard) requires CVA be recorded in statements
- Basel III extended capital framework to include a CVA charge

\(^{14}\) eg. Duffie and Canabarro ('04) and Gregory ('15).
What is CVA?

- Counterparty credit risk $\implies$ "credit valuation adjustment"\(^{15}\) or CVA
- Basically upfront cost of dynamically hedging counterparty default
- Must be added to classical value, or cooked into deal payments

Credit Valuation Adjustment

$$CVA(t) = \mathbb{E}_t \left[ \int_t^T \text{Discounted Counterparty Loss}(s) \, ds \right]$$

- IFRS13 (accounting standard) requires CVA be recorded in statements
- Basel III extended capital framework to include a CVA charge

\(^{15}\) eg. Duffie and Canabarro ('04) and Gregory ('15).
What is CVA?

- Counterparty credit risk $\implies$ “credit valuation adjustment”\textsuperscript{16} or CVA
- Basically upfront cost of dynamically hedging counterparty default
- Must be added to classical value, or cooked into deal payments

Credit Valuation Adjustment

\[
CVA(t) = \mathbb{E}_t\left[\int_t^T e^{-\int_t^s r(u) du} \lambda(s)(V(s))^+ ds\right]
\]

- IFRS13 (accounting standard) requires CVA be recorded in statements
- Basel III extended capital framework to include a CVA charge

\textsuperscript{16}eg. Duffie and Canabarro ('04) and Gregory ('15).
What is CVA?

- Counterparty credit risk $\Rightarrow$ "credit valuation adjustment"\(^{17}\) or CVA
- Basically upfront cost of dynamically hedging counterparty default
- Must be added to classical value, or cooked into deal payments

**Credit Valuation Adjustment**

$$CVA(t) = \mathbb{E}_t \left[ \int_t^T e^{-\int_t^s r(u) \, du} \lambda(s)(V(s))^+ \, ds \right]$$

- IFRS13 (accounting standard) requires CVA be recorded in statements
- Basel III extended capital framework to include a CVA charge

\(^{17}\) eg. Duffie and Canabarro ('04) and Gregory ('15).
Crucial to note that CVA is computed at the counterparty level.

Assume swaps with values $V_1^{\text{swap}}$ and $V_2^{\text{swap}}$ with counterparty.

Portfolio exposure is not the sum of individual swap exposures.

Portfolio Exposure Aggregation

$$(V^{\text{portfolio}})^+ = (V_1^{\text{swap}} + V_2^{\text{swap}})^+ \neq (V_1^{\text{swap}})^+ + (V_2^{\text{swap}})^+$$

Clearly $V_2^{\text{swap}} < 0$ could offset $V_1^{\text{swap}} > 0$, and vice versa.

Thus the incremental CVA involves whole counterparty portfolio.

Why xVA calcs are so expensive... some xVAs involve many c/parties!
A Practical CVA Computation

- Evaluate the CVA formula via discretization and Monte Carlo
- Use \( p = 1, \ldots, P \) paths and \( i = 0, \ldots, T \) timesteps

CVA Calculation

\[
CVA(t_0) = \frac{1}{P} \sum_{i=1}^{T} \sum_{p=1}^{P} \lambda_p(s_i) e^{-\int_{t}^{s_i} r_p(u) \, du} (V_p(s_i))^+ \Delta s_i
\]

1. Simulate paths state variables \( X_{p,i} \), e.g., rates, hazards, FX
2. Revalue portfolio along all paths \( V_{p,i}^{\text{pf} \text{folio}} = V_{1, p, i}^{\text{swap}} + \cdots \)
3. Evaluate \((V_{p,i})^+\), weight by discount factors & hazards and aggregate
A Practical CVA Computation

- Evaluate the CVA formula via discretization and Monte Carlo
- Use $p = 1, \ldots, P$ paths and $i = 0, \ldots, T$ timesteps

CVA Calculation

$$
CVA(t_0) = \frac{1}{P} \sum_{i=1}^{T} \sum_{p=1}^{P} \lambda_p(s_i) e^{-\int_{t}^{s_i} r_p(u) du} (V_p(s_i))^+ \Delta s_i
$$

1. Simulate paths state variables $X_{p,i}$, eg. rates, hazards, FX
2. Revalue portfolio along all paths $V_{p,i}^{\text{portfolio}} = V_{1,p,i}^{\text{swap}} + \cdots$
3. Evaluate $(V_{p,i})^+$, weight by discount factors & hazards and aggregate
Figure: Example of simulations involved in a CVA calculation. Portfolio consists of a 5Y swap and a 10Y swap.
Figure: Example of simulations involved in a CVA calculation. Portfolio consists of a 5Y swap and a 10Y swap.
Figure: Example of simulations involved in a CVA calculation. Portfolio consists of a 5Y swap and a 10Y swap.
**Figure:** Example of simulations involved in a CVA calculation. Portfolio consists of a 5Y swap and a 10Y swap.
Figure: Example of simulations involved in a CVA calculation. Portfolio consists of a 5Y swap and a 10Y swap.
Figure: Example of simulations involved in a CVA calculation. Portfolio consists of a 5Y swap and a 10Y swap.
Main Challenges in CVA

- Portfolios can span multiple risk factors, *e.g.* swaps in many CCYs.
- Building models is difficult and building *hybrid* models is more so.
- Require correlation structures for (latent) variables & joint calibration.
- Computing $V_{p,i}$ can be an $O(P^2)$ problem for exotics.
- In practice this can be extremely expensive and we try to avoid.
- American MC (AMC) is a powerful regression-based $O(P)$ algorithm.
Etymology from its use in pricing American (early-exercise) options

Idea is to use the original $P$ paths to build future values via projection

**Future Values by Regression**

\[ V(t) = \tilde{E}_t [C(T)] \implies \]

\[ C(X(T)) = V(X(t)) + \epsilon(t, T), \tilde{E}_t[\epsilon(t, T)] = 0 \]

\[ y = f(x) + \nu, \mathbb{E}[\nu] = 0 \]

Evaluated via regression with \( f(\cdot) = \beta_0 \phi_0(\cdot) + \beta_1 \phi_1(\cdot) + \cdots \)

Selection of basis functions $\phi(\cdot)$ crucial, especially in high dimensions

Difficult to implement for generic products with scripted payoffs
American Monte Carlo

- Etymology from its use in pricing American (early-exercise) options
- Idea is to use the original $P$ paths to build future values via projection

Future Values by Regression

$$V(t) = \tilde{E}_t [C(T)] \implies C(X(T)) = V(X(t)) + \epsilon(t, T), \tilde{E}_t[\epsilon(t, T)] = 0$$

$$y = f(x) + \nu, \ E[\nu] = 0$$

- Evaluated via regression with $f(\cdot) = \beta_0 \phi_0(\cdot) + \beta_1 \phi_1(\cdot) + \cdots$
- Selection of basis functions $\phi(\cdot)$ crucial, especially in high dimensions
- Difficult to implement for generic products with scripted payoffs
American Monte Carlo

- Etymology from its use in pricing American (early-exercise) options
- Idea is to use the original $P$ paths to build future values via projection

Future Values by Regression

\[ V(t) = \widetilde{E}_t [C(T)] \implies \]
\[ C(X_p(T)) = V(X_p(t)) + \epsilon_p(t, T), \widetilde{E}_t[\epsilon_p(t, T)] = 0 \]
\[ y = f(x) + \nu, \ E[\nu] = 0 \]

- Evaluated via regression with $f(\cdot) = \beta_0 \phi_0(\cdot) + \beta_1 \phi_1(\cdot) + \cdots$
- Selection of basis functions $\phi(\cdot)$ crucial, especially in high dimensions
- Difficult to implement for generic products with scripted payoffs
American Monte Carlo

- Etymology from its use in pricing American (early-exercise) options
- Idea is to use the original $P$ paths to build future values via projection

**Future Values by Regression**

\[ V(t) = \widehat{E}_t \left[ C(T) \right] \implies \\
C(X_p(T)) = V(X_p(t)) + \epsilon_p(t, T), \quad \widehat{E}_t[\epsilon_p(t, T)] = 0 \]

\[ y = f(x) + \nu, \quad \mathbb{E}[\nu] = 0 \]

- Evaluated via regression with \( f(\cdot) = \beta_0 \phi_0(\cdot) + \beta_1 \phi_1(\cdot) + \cdots \)
- Selection of basis functions \( \phi(\cdot) \) crucial, especially in high dimensions
- Difficult to implement for generic products with scripted payoffs

Andrew McClelland, Numerix
American Monte Carlo

- Etymology from its use in pricing American (early-exercise) options
- Idea is to use the original \( P \) paths to build future values via projection

**Future Values by Regression**

\[
V(t) = \widehat{E}_t [C(T)] \implies \\
C(X_p(T)) = V(X_p(t)) + \epsilon_p(t, T), \quad \widehat{E}_t[\epsilon_p(t, T)] = 0 \\
y_p = f(x_p) + \nu_p, \quad \mathbb{E}[\nu_p] = 0
\]

- Evaluated via regression with \( f(\cdot) = \beta_0\phi_0(\cdot) + \beta_1\phi_1(\cdot) + \cdots \)
- Selection of basis functions \( \phi(\cdot) \) crucial, especially in high dimensions
- Difficult to implement for generic products with scripted payoffs
Figure: Example of AMC for a simple Bermudan swaption. Pathwise cashflows are projected onto the short rate using a Hull White ('90) model.
American Monte Carlo

- Etymology from its use in pricing American (early-exercise) options
- Idea is to use the original $P$ paths to build future values via projection

Future Values by Regression

$$V(t) = \tilde{E}_t [C(T)] \implies$$

$$C(X_p(T)) = V(X_p(t)) + \epsilon_p(t, T), \tilde{E}_t[\epsilon_p(t, T)] = 0$$

$$y_p = f(x_p) + \nu_p, \mathbb{E}[^\nu_p] = 0$$

- Evaluated via regression with $f(\cdot) = \beta_0 \phi_0(\cdot) + \beta_1 \phi_1(\cdot) + \cdots$
- Selection of basis functions $\phi(\cdot)$ crucial, especially in high dimensions
- Difficult to implement for generic products with scripted payoffs
**Figure:** Example of a Numerix payoff script for a Bermudan swaption. All DISCOUNTING variables are subjected to a regression as the algorithm works backward in time.
Where Does Matlab Come In? 1

- Clients using Numerix have their own views on how to aggregate
- Instead of writing NX scripts & functions to do the maths use Matlab
- Expose intermediate calcs to allow user logic
- Many assumptions on counterparty behavior and default correlations
  - eg. “wrong-way risk” or WWR: exposures correlate with default prob
- Subjective and clients want control over default prob trajectories
Also complications re. collateral arrangements and margining

- Ratings triggers can force lower collateral thresholds and min transfers
- Early-terminations as function of market conditions or credit health?
- Similarly so for rolling of shorter-term trades
- Collateral rehypothecation across counterparties for funding offsets
The really difficult calculations relate to portfolio-level effects.

Looking for capital impacts (KVA) and initial margin impacts (MVA).

Capital and initial margin requirements are becoming more onerous\(^ {18} \)

Pricing these and/or optimizing against them is crucial.

Also benefits to fully tracking cost of collateral transformation.

\(^{18}\) See eg. Basel FRTB (BCBS365) and Basel-IOSCO (BCBS317).