Modelling Hedge Fund Returns Using State Space Models
Firm Overview
Stenham Asset Management
25-year award winning performance

» Pioneering hedge fund investment specialists since 1980s

» US$ 2billion AUM

» 37 dedicated employees

» CIO 23 years investment experience, 17 years with Stenham

» Alignment of interest: the Stenham team are significant co-investors

» Consistently won industry recognition over many years

» Authorised and regulated by the FCA, SEC, GFSC and FSB
Industry Leading Quantitative Analysis & Risk Systems
Where MATLAB fits within our risk and quantitative analysis systems

- **C*NEO**
  - Data Management & Reporting
  - Robust SQL based database solution for data storage and handling
  - Flexible Excel add-in analytics
  - Ideal for customised reporting

- **RiskData**
  - Off-the-Shelf, Returns-Based Risk System
  - Robust "polymodel" approach to factor analysis and risk management
  - Long-Term historical factor based Monte Carlo simulation
  - Conservative "StressVAR" estimates

- **AlternativeSoft**
  - Off-the-Shelf Data Analysis System
  - Flexible and responsive multi-factor approach
  - Highly interactive, intuitive and configurable
  - Modular design mirroring stages in the investment process
  - Intuitive graphical output

- **RiskMetrics**
  - Off-the-Shelf, Position-Based Risk System
  - Position-based portfolio transparency
  - Underlying portfolios updated on a monthly basis
  - Best tool for stress testing of current portfolios

- **MATLAB**
  - Data Analysis and Application Development
  - Technical, maths-based programming language
  - Extensive libraries for financial analysis
  - Ideal environment for developing proprietary models

www.stenhamassetmanagement.com
Traditional ways of modelling hedge fund returns
Traditional models for hedge fund returns

Basic linear factor models

\[ y_t = \alpha + \sum_{j=1}^{k} \beta_j x_{j,t} + \varepsilon_t \]
Traditional models for hedge fund returns

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Basic linear factor models: what factors to use?

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Traditional models for hedge fund returns

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- CAPM (S&P)
- Fama-French (HML, SMB)
- Carhart (Momentum)
Traditional models for hedge fund returns

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- CAPM (S&P)
- Fama-French (HML, SMB)
- Carhart (Momentum)
- Fung and Hsieh (2001)
Traditional models for hedge fund returns

(At least) three problems with basic linear factor models

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Traditional models for hedge fund returns

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- Non-linearity
Traditional models for hedge fund returns

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- Non-linearity → non-linear factors
Traditional models for hedge fund returns

(At least) three problems with basic linear factor models

\[ y_t = \alpha + \sum_{j=1}^{k} \beta_j x_{j,t} + \varepsilon_t \]

- Non-linearity
- Time dependency
Traditional models for hedge fund returns

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- Serial correlation
Traditional models for hedge fund returns

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\[ y_t = \alpha + \sum_{j=1}^{k} \beta_j x_{j,t} + \varepsilon_t \]

- Non-linearity
- Time dependency
- Serial correlation \( \Rightarrow AR(p) \)
Traditional models for hedge fund returns

Some of the questions we wanted to investigate

- Does formally allowing for time-varying parameters improve the performance of the factor models?
- Does it result in more stable parameters?
- Does inclusion of an autoregressive term provide a better fit on average?
- How does that compare with adding additional factors?
- How well do out-of-sample models perform relative to in-sample ones?
- Do the results vary across different hedge fund strategies?
- How much harder are these models to implement than a rolling window regression?
A brief introduction to State Space Models (SSM)
Introduction to State Space Models

The general form for univariate Gaussian State Space Models

\[ y_t = Z_t a_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma^2) \]
\[ a_{t+1} = T_t a_t + R_t \eta_t, \quad \eta_t \sim NID(0, Q_t) \]

- \( y_t \equiv \text{observed time series (} t = 1, \ldots, n \)\)
- \( a_t \equiv (m \times 1) \text{ unobserved state variables} \)
- \( Z_t \equiv (m \times 1) \text{ observation vector} \)
- \( \varepsilon_t \equiv \text{observation error (variance } \sigma^2 \)\)
- \( T_t \equiv (m \times m) \text{ transition matrix} \)
- \( R_t \equiv (m \times r) \text{ selection matrix} \)
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Introduction to State Space Models

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Introduction to State Space Models

Example 1: the local level model

\[ y_t = Z_t' a_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2) \]

\[ a_{t+1} = T_t a_t + R_t \eta_t, \quad \eta_t \sim NID(0, Q_t) \]

\[ a_t = \mu_t, \quad \eta_t = \xi_t, \quad Z_t = T_t = R_t = 1, \quad Q_t = \sigma_\xi^2 \]

\[ y_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2) \]

\[ \mu_{t+1} = \mu_t + \xi_t, \quad \xi_t \sim NID(0, \sigma_\xi^2) \]
Introduction to State Space Models
Estimation of State Space Models using the Kalman filter and smoother

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Source: Commandeur and Koopman (2007)
Introduction to State Space Models

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1. Start at \( t = 1980 \)

Source: Commandeur and Koopman (2007)
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1. Start at \( t = 1980 \)

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   you can use that to update \( a_{t+1} \)

5. You do that by adjusting \( a_t \) by \( K_t(y_t - a_t) \)
   where \( K_t \alpha \frac{\sigma^2}{\sigma^2} \)

Source: Commandeur and Koopman (2007)
Introduction to State Space Models

Estimation of State Space Models using the Kalman filter and smoother

- Predicted state variables:
  \[ \hat{a}_t | (y_1, \ldots, y_{t-1}; x_1, \ldots, x_{t-1}) \]

- Filtered state variables:
  \[ \hat{a}_t | (y_1, \ldots, y_t; x_1, \ldots, x_t) \]

- Smoothed state variables:
  \[ \hat{a}_t | (y_1, \ldots, y_n; x_1, \ldots, x_n) \]
Testing and comparing models for hedge fund returns
Testing and comparing models
A very basic test: how accurately can we estimate funds’ market beta?

Four models, four estimation methodologies, ~6,500 funds:

Model 1: \[ y_t = \alpha_t + \beta_t x_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma^2_\varepsilon) \]

Model 2: \[ y_t = \alpha_t + \beta_t x_t + \phi_t y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma^2_\varepsilon) \]

Model 3: \[ y_t = \alpha_t + \beta_t x_t + \gamma_t z_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma^2_\varepsilon) \]

Model 4: \[ y_t = \alpha_t + \beta_t x_t + \gamma_t z_t + \phi_t y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma^2_\varepsilon) \]

Estimation method 1: 12mo rolling windows (out of sample)

Estimation method 2: Kalman filter (out of sample)

Estimation method 3: static linear regression (in sample)

Estimation method 4: Kalman smoother (in sample)
Testing and comparing models

Results for a sample fund – Model 2 – 12mo rolling window
Testing and comparing models

Results for a sample fund – Model 2 – Kalman filter

Components

Market beta

AR(1)

Residuals
Testing and comparing models

Results for a sample fund – Model 2 – Kalman smoother

![Graphs showing components, market beta, AR(1), and residuals over time with confidence intervals.]
Testing and comparing models

Results for a sample fund: comparing across models and estimation techniques

Cond. Forecast ≡ \{\hat{y}_t | \hat{\alpha}_t; \hat{\beta}_t; \hat{\phi}_t; \hat{x}_t; \hat{z}_t; y_{t-1}\} = y_t - \varepsilon_t
## Testing and comparing models

Results across ~6,500 funds: median R$^2$ of conditional forecast

<table>
<thead>
<tr>
<th></th>
<th>Equities</th>
<th>Eq. + AR(1)</th>
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Results across ~6,500 funds: median $R^2$ of conditional forecast

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Results across ~6,500 funds: median R² of conditional forecast

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## Testing and comparing models

Results across ~6,500 funds: percentage of funds with higher $R^2$

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Testing and comparing models

Results across ~6,500 funds: median standard deviation of beta

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<td>13.6%</td>
<td>15.0%</td>
<td>10.9%</td>
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<tr>
<td>Static Linear Regression (IS)</td>
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<tr>
<td>Kalman Smoother (IS)</td>
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Testing and comparing models

Results across HF strategies: median adjusted R\(^2\) of conditional forecast

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<th>Hedge Fund Strategy</th>
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<th>Kalman Filter (OOS)</th>
<th>Static Linear Regression (IS)</th>
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<tr>
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<tr>
<td>Top-Down</td>
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<td>8.4%</td>
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<tr>
<td>Value</td>
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<td>10.6%</td>
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</tr>
</tbody>
</table>
Conclusion
Conclusion

Some of the questions we wanted to investigate

- Does formally allowing for time-varying parameters improve the performance of the factor models?
- Does it result in more stable parameters?
- Does inclusion of an autoregressive term provide a better fit on average?
- How does that compare with adding additional factors?
- How well do out-of-sample models perform relative to in-sample ones?
- Do the results vary across different hedge fund strategies?
- How much harder are these models to implement than a rolling window regression?
Conclusion
Some of the questions we wanted to investigate

• Does formally allowing for time-varying parameters improve the performance of the factor models? YES
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- Does it result in more stable parameters? YES
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- How does that compare with adding additional factors? Depends
- How well do out-of-sample models perform relative to in-sample ones?
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- Does formally allowing for time-varying parameters improve the performance of the factor models? **YES**
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- How well do out-of-sample models perform relative to in-sample ones? **So-so**
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- Does formally allowing for time-varying parameters improve the performance of the factor models? **YES**
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- How much harder are these models to implement than a rolling window regression?
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Some of the questions we wanted to investigate

- Does formally allowing for time-varying parameters improve the performance of the factor models? **YES**
- Does it result in more stable parameters? **YES**
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- How well do out-of-sample models perform relative to in-sample ones? **So-so**
- Do the results vary across different hedge fund strategies? **Not really**
- How much harder are these models to implement than a rolling window regression? **Not much**
Conclusion

Advantages of using a State Space Modelling framework

• Extremely flexible modelling framework.
• Easily incorporates time-varying parameters.
• Dynamic factor estimates are smoother and more reliable than those from rolling windows (at least for HF returns).
• Both in-sample and out-of-sample testing can be implemented without resource-intensive loops.
• Linear Gaussian models can be efficiently estimated using fast, efficient recursive closed-form techniques such as the Kalman filter and smoother.
• Less restrictive models (non-linear/non-Gaussian) can be estimated using modified filtering techniques or with numerical methods.
• Recursive estimation makes it easy to simulate and forecast from estimated models as well as to deal with missing data points.
• Intuitive link with Bayesian statistical methods and econometrics.
• MathWorks® and third-party SSM toolboxes readily available.
Conclusion

Advantages of using MATLAB

- MATLAB Desktop environment allows for fast and easy interactive data analysis and model research and development.
- Distinct advantages of having integrated computational and application development environments.
- Wide availability of toolboxes and third-party libraries.
- Extensive plotting and visualisation tools.
- Thorough documentation and extensive/flexible support.
- Ability to integrate with many other environments (Excel, Access, SQL, etc.).
- Ability to compile and distribute packaged applications.
- One-off product license with reasonably priced maintenance costs.
Modelling Hedge Fund Returns Using State Space Models

References

Books:


Other:


Appendix – Extra Slides
Traditional models for hedge fund returns

Modelling serial correlation and illiquidity in HF returns (Getmansky, Lo and Makarov, 2004)

\[ y_t = \alpha + \sum_{j=1}^{k} \beta_j x_{j,t} + \varepsilon_t, \quad E[x_{j,t}], E[\varepsilon_t] = 0, \quad \varepsilon_t, x_{j,t} \sim \text{IID} \]

\[ y_t^0 = \theta_0 y_t + \theta_1 y_{t-1} + \cdots + \theta_p y_{t-p} \]

\[ \theta_j \in [0,1], \quad j = 0, \ldots, p \]

\[ \sum \theta_j = 1 \Rightarrow E[y_t^0] = E[y_t] \]
Introduction to State Space Models

Example 2: a local level model with a single static market factor

\[ y_t = Z_t' a_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma^2_\varepsilon) \]

\[ a_{t+1} = T_t a_t + R_t \eta_t, \quad \eta_t \sim NID(0, Q_t) \]

\[ a_t = (\alpha_t', \beta_t'), \quad \eta_t = (\xi_t', 0), \quad T_t = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad Z_t = \begin{bmatrix} 1 \\ x_t \end{bmatrix}, \]

\[ R_t = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad Q_t = \begin{bmatrix} \sigma^2_\xi & 0 \\ 0 & 0 \end{bmatrix} \]

\[ y_t = \alpha_t + \beta_t x_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma^2_\varepsilon) \]

\[ a_{t+1} = \alpha_t + \xi_t, \quad \xi_t \sim NID(0, \sigma^2_\xi) \]

\[ \beta_{t+1} = \beta_t \]
Introduction to State Space Models

Example 3: a local level model with a single time-varying market factor

\[
y_t = Z_t' a_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma_{\varepsilon}^2)
\]

\[
a_{t+1} = T_t a_t + R_t \eta_t, \quad \eta_t \sim NID(0, Q_t)
\]

\[
a_t = \begin{pmatrix} \alpha_t \\ \beta_t \end{pmatrix}, \quad \eta_t = \begin{pmatrix} \xi_t \\ \zeta_t \end{pmatrix}, \quad T_t = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad Z_t = \begin{pmatrix} 1 \\ x_t \end{pmatrix},
\]

\[
R_t = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad Q_t = \begin{bmatrix} \sigma_{\xi}^2 & 0 \\ 0 & \sigma_{\zeta}^2 \end{bmatrix}
\]

\[
y_t = \alpha_t + \beta_t x_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma_{\varepsilon}^2)
\]

\[
\alpha_{t+1} = \alpha_t + \xi_t, \quad \xi_t \sim NID(0, \sigma_{\xi}^2)
\]

\[
\beta_{t+1} = \beta_t + \zeta_t, \quad \zeta_t \sim NID(0, \sigma_{\zeta}^2)
\]
Introduction to State Space Models

Example 4: an AR(2) model

\[ y_t = Z_t'a_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2) \]
\[ a_{t+1} = T_t a_t + R_t \eta_t, \quad \eta_t \sim NID(0, Q_t) \]

\[ a_t = \begin{pmatrix} y_t \\ \phi_2 y_{t-1} \end{pmatrix}, \quad Z_t = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \]
\[ T_t = \begin{bmatrix} \phi_1 & 1 \\ \phi_2 & 0 \end{bmatrix}, \quad R_t = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \]
\[ \eta_t = \begin{pmatrix} \xi_{t+1} \\ 0 \end{pmatrix}, \quad Q_t = \begin{bmatrix} \sigma_\xi^2 & 0 \\ 0 & 0 \end{bmatrix}, \quad \sigma_\varepsilon^2 = 0 \]

\[ y_{t+1} = \phi_1 y_t + \phi_2 y_{t-1} + \xi_{t+1} \]
\[ \xi_t \sim NID(0, \sigma_\xi^2) \]