

The Towers of Hanoi as a Cyber-Physical System Education Case Study

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Abstract—While extensive use of software in technical systems necessitates a multi-domain systems perspective education has increasingly focused on specific fields of engineering science. Project-based learning (PBL) attempts to overcome this disconnect with authentic problem solving at the core of the approach. A medium-sized Cyber-physical system (CPS) is presented that includes sufficient detail to highlight pertinent CPS problems and direct students to multi-domain authentic problems.

I. INTRODUCTION

The past half century has witnessed astonishing advances along two separate but related axes: (i) miniaturization has rendered computing power a commodity and (ii) computation has become a full fledged discipline based on solid fundamentals. These combined advances have come to drive many of the technologies that are ubiquitous today. For example, network connectivity relies heavily on computational communication protocols that operate reliably, at low power, and with a miniature form factor. As another example, multi-purpose mobile devices and automobiles derive much of their feature suite from software configurability.

Not only has software rapidly become the technology of choice because of its functional flexibility, software applications have also pervaded the very design of technical systems. For example, Model-Based Design relies heavily on an electronic representation of design artifacts which allows transformative design models based on computational simulation for design space exploration, automatic code generation, specification testing, and full design traceability [1], [2].

With the terrific advantages to network and information technology, the challenge is that technical systems are becoming highly connected and interacting, often in manners and modalities that are difficult to analyze let alone foresee. As a consequence, system integration has become an increasingly demanding (often artisan) task. As long as there is a single original equipment manufacturer (OEM) responsible for the system as an end product, system integration can proceed in an organized manner and controlled environment. An important element of this paradigm is that there is a clear boundary between the system and its environment, where the environment is often treated as a disturbance that the system ought to reject as much as possible (e.g., such as in control theory [3]).

More recently, however, systems have started coming on-line with automated functionality that is not subject to the

responsibility of a single OEM and where it is necessary to exhibit measured interaction with a more structured environment. For example, an autonomous machine (e.g., a rescue robot) may rely on image processing technology for its situational awareness that enables it to not only react but also interact with the environment. In even more advanced scenarios, systems may connect and communicate at an information level to collaborate and implement a common functionality as a shared objective. For example, in future vehicle-to-infrastructure scenarios an automobile may be equipped with a feature to communicate with a traffic light and autonomously decide when to proceed or when to stop. Such information level interaction is beyond today's physical observation by a traffic light of an approaching emergency vehicle and turning the light green in response.

Systems that implement their functionality with a distinct network and information component increasingly rely on a corresponding infrastructure to access a possibly global information space. When deployed to operate in and interact with a physical environment, these systems have come to be known as *cyber-physical systems* [4]. While the underlying technology of cyber-physical systems (CPS) is of a networked and embedded systems nature, CPS as a *paradigm* requires novel and transformative methodologies and technologies [5].

Turning to engineering education, the developments that spawned the field of CPS have a profound impact because of the foundational, and, therefore, inherently multidisciplinary nature. This impact is particularly dramatic because of a contemporary trend to increasingly focus higher education on engineering science in specialized disciplines. As attested to by the *conceive, design, implement, and operate* (CDIO) framework for engineering education [6], over the past half century the curriculum for engineering students has included ever less attention to system building skills in favor of ever more domain specific theory [7].

The principles of CDIO are an attempt to reverse this trend in education because combined with the trend in technical systems design a distinct disconnect has emerged between the needs in industry and the skill set of graduating engineers. The situation is further exacerbated because while research universities may emphasize the need for multidisciplinary faculty involvement, they are susceptible to tacitly discouraging such involvement instead. As long as faculty members

operate within the confines of a specialized field, all the facilities, incentives, merits, and opportunities exist to further their careers. Operating beyond these confines is a high-risk endeavor that is likely to be impeded from many directions yet is essential to creating the provisions for a flourishing CPS ecosystem across industry and academia.

But all is not gloomy. At least in education, *project-based learning* (PBL) has gained increasing popularity (e.g., [8], [9]). Similar to CDIO, PBL strives to redirect attention to engineering skills such as design, manufacture, systems, communication, critical thinking, creativity, and teamwork. Though still far from universally adopted, PBL is quickly spreading globally among colleges and universities. Based on the premise that students are well served by tackling a project in a comprehensive yet in-depth manner, PBL requires five features to be present [8] such that the project:

- is central to the teaching strategy and is reflected as the main element in the curriculum;
- brings out the desired questions and learning experiences about the specific domains and their principles as opposed to only an academic exercise;
- actively develops new knowledge in a constructive manner;
- facilitates autonomous exploration by the students; and
- is authentic, implementable, and proposes to solve a problem that exists outside of the classroom.

Similar to CDIO, PBL projects do not have a single correct answer, as opposed to what much of engineering education had become at the turn of the millennium. Instead, PBL encourages exploration along many different avenues where the corresponding inquiry aligns with *intentional learning* models that support constructive knowledge building [10].

Often times, PBL projects are hardware-oriented and may be difficult to facilitate in a general classroom setting. This paper leverages information technology to provide students with a computational model of a reasonably complex system where sufficient fidelity is modeled so as to be authentic (as a key element of PBL [8]) and to bring out many of the issues found in the design of CPS. The availability of an operational complex system helps scaffold the learning experience of students by allowing them to quickly learn about the complex mechanisms, interactions between various mechanisms, expert design methods, etc., that are central to a discipline. Moreover, the scaffolding enables learning in an environment where knowledge is likely to be applied, which from a *situated cognition* or *anchored instruction* perspective is much preferred over presenting learning material void of context [8], [11], [12]. Empirical studies have shown that a high fidelity virtual environment can serve as a successful surrogate to practical experiments [13].

In the proceedings, Section II first introduces a CPS case study of authentic complexity. In this case study, Section III highlights a number of features as starting points for exploratory study that drives students to central networked embedded systems aspects of CPS. Section IV then illustrates how the case study drives students to an exploration of multi-domain issues. Section V presents conclusions of the work.



Fig. 1. Towers of Hanoi as a SCADA System

II. TOWERS OF HANOI AS A CYBER-PHYSICAL SYSTEM CASE STUDY

A CPS case study of medium complexity is conceived as a manufacturing facility that solves the Towers of Hanoi puzzle.¹ As a supervisory control and data acquisition (SCADA) system such a facility may consist of a pick and place machine that is programmed to move between three locations where it either picks or places a block (see Fig. 1). In a conventional systems design paradigm there may be one OEM responsible for delivering the system with specifications as to where the stack of blocks is initially located, how many blocks there are, in what order the blocks are stacked, and what the resulting order of the stacked blocks should be. The machine may then be programmed by sequence control to await an operator start command along with information about the initial stack ordering of blocks after which a series of pick and place operations are executed to obtain the desired order. Before delivery, a series of acceptance tests by the OEM determine whether the system performs satisfactorily for each of the possible order permutations of the initial stack.

In a CPS paradigm, the Towers of Hanoi becomes a system of communicating systems that may each have their own OEM. For example, instead of an operator initiating a particular sequence of actions based on observing the initial stack ordering, the blocks may become actively involved in the sorting process as ‘smart blocks’. Each of the blocks may obtain sensory information about its whereabouts, potentially aided by communicating with blocks that are near. Based on their situational awareness, the blocks may then communicate the operations they should be subjected to the pick and place machine. Each of the blocks may implement an individual plan and the machine merges these different plans into an overall behavior of the stack of blocks. If successful, the requested control operations based on each of the local plans result in the *emerging behavior* of a particular stack ordering.

III. TEACHING OPPORTUNITIES IN THE CASE STUDY

As a CPS case study for engineering education, the distributed Towers of Hanoi embodies a broad range of topics across various fields and domains in the discipline of engineering. To enable convenient access to the educational experience, a fully functioning Simulink® [14] model (see Fig. 2) has been developed and made publicly available.²

¹https://en.wikipedia.org/wiki/Tower_of_Hanoi

²The model can be downloaded from the MATLAB® Central File Exchange (<http://www.mathworks.com/matlabcentral/fileexchange/38515>).

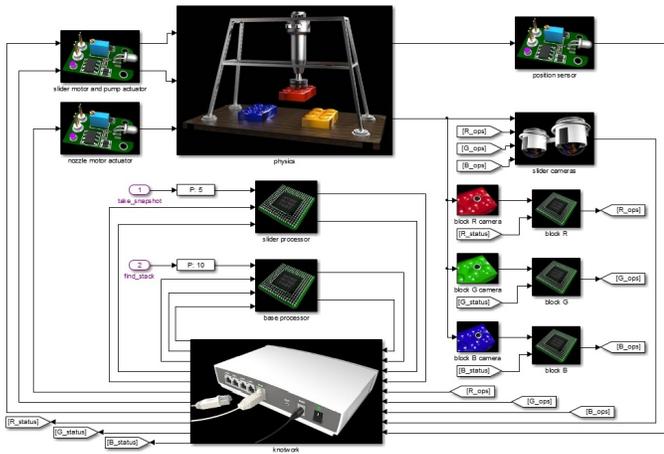


Fig. 2. Simulink[®] Model of the Distributed Towers of Hanoi

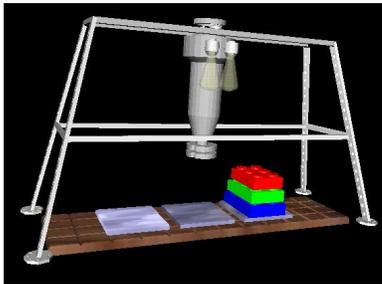


Fig. 3. The Synthesized Towers of Hanoi Scene

A. Image Processing

In a CPS setting, the pick and place machine must first develop a situational awareness, reflected by an initially unknown location of the stack of blocks. To this end, a sweep mode that precedes any pick and place action employs a video stream of the area underneath the slider that moves the pick and place nozzle in a horizontal range. More specifically, two cameras provide a stereoscopic view from which it can be determined at which location of the slider there is an object closer to the camera. This object is then interpreted as the stack of blocks.

A virtual representation (see Fig. 3) makes it possible to synthesize two video streams from the cameras. As such, the stereopsis algorithm can be developed and tested in a controlled setting without interferences in a physical environment (lighting of the scene, shades of artifacts and people around the scene, differences in camera output, etc.) which can quickly overwhelm a novice. Moreover, in a synthetic environment experiments can often be run faster than the physical counterpart, which makes for a more satisfying learning experience, especially when developing an elaborate plan.

B. Control

Though not exclusive to CPS, control is an essential element of most all CPS. The Towers of Hanoi facilitate experimenting with both feedforward and feedback control.

1) *Feedback Control*: The physics of the pick and place machine is modeled to have a *dc* motor move the slider in the horizontal direction. The input to the motor is a control voltage

that corresponds to a force exerted by the motor, which in turn causes an acceleration of the slider. In the case study, first the system is linearized around an operating point away from 0 velocity. The control law is then based on the linear model by a Gaussian regulator with output feedback [3].

Because of the discrepancy between the physics and the linearized model away from 0 velocity, the feedback control around 0 velocity is controlled by an alternate control scheme. In this case, the stiction effect that is included in the nonlinear physics models requires a minimum force that is larger than the breakaway stiction force. Bang-bang control with output either the negative or positive minimum force ensures that the slider position is fine-tuned after the Gaussian control based on the linear model is out of its domain of validity.

2) *Feedforward Control*: Once the slider is positioned in the horizontal direction, a nozzle moves in the vertical direction to perform the picking and placing of a block. The vertical motion must be coordinated with operating a pneumatic pump that creates an airflow through the nozzle and around a block. When the nozzle is sufficiently near, the airflow creates a (negative) pressure that pulls a block against the nozzle.

The vertical motion of the nozzle is also controlled by a modeled *dc* motor. However, to enable fast up and down motion (e.g., a surface mount device must complete a pick and place operation in tens of milliseconds) feedforward control determines the control force. Feedforward control is enabled because of an assumed fixed height of each block and by some bookkeeping to keep track of the top of the stack in each of the three locations as blocks are being moved around. The feedforward control can be synthesized, for example, based on model checking technology [2], [15], [16].

C. Distributed Control

As each of the ‘smart blocks’ devise their individual plans with corresponding control actions, the overall CPS comprises a number of interacting local controls. Here, a broad range of issues must be addressed such as prioritization between the pick and place service requests by the various blocks (with dynamically changing priorities), synchronization between the machine action and the block planning state, and correctly interacting behavior to ensure the desired emerging behavior.

D. Multirate Embedded Systems

Overall, the distributed Towers of Hanoi CPS embodies a number of activities that execute at different rates: (i) the control loops require a fast sample rate and execute with a 5 ms period, (ii) the pick and place service requests by the blocks are less critical and execute with a 20 ms period, and (iii) the processing of video stream images is computationally intensive and executes with a 100 ms period. The result is a multirate embedded system with distinct complications when rate transitions are necessary. For example, the stereopsis analysis of the video stream finds the point in time of the video stream where the stack of blocks is observed by the cameras. Once the analysis determines that the stack has been found, the current location of the slider may be recorded as such. However, the location is measured at a rate of 5 ms while the video stream is analyzed at a rate of 100 ms. With only one deterministic rate transition, this already results in a location

measurement that is about 100 ms later in time. Depending on how fast the feedback control of the slider is, this may cause a significant error in the recorded location of the stack of blocks.

IV. MULTI-DOMAIN DIRECTION OF INQUIRY

One of the values of a case study such as the distributed Towers of Hanoi is that it directs students to quickly learn how, in complex systems, there are many interactions across features and how this calls for a true multi-domain *systems* view rather than isolated consideration of a single feature.

For example, the stereopsis analysis of the video stream relies on careful calibration to determine the detection of the stack location based on sophisticated image processing. Likewise, the slider feedback control that switches between a Gaussian regulator and bang-bang control must be carefully calibrated so as to preserve the properties derived based on a linear model when applied to the full nonlinear model with stiction effects. There is, however, a delicate interaction between the stereoscopic vision and the slider positioning features. For example, if the feedback control moves the slider faster, the video stream receives less detail of the scene when processing 10 frames per second (because of the 100 ms sample period). Thus the stereopsis analysis may fail if the slider moves faster and recalibration may be required.

Much like the error in the recorded location of the stack of blocks as discussed previously, in a conventional design paradigm, this recalibration issue is caught and resolved during system integration. However, in a CPS paradigm, these issues arise and must be tackled after a system has been deployed. This calls for automated novel functionality idiosyncratic to CPS. A sufficiently complex case study such as the distributed Tower of Hanoi, implemented at a systems level perspective, helps students identify concretely where systems integration may arise and allows experimenting with various solution approaches. Moreover, as a standardized benchmark, such a case study can even serve to compare and contrast different proposed solutions by various student teams.

V. CONCLUSION

Extensive use of software in technical systems has made a multi-domain systems perspective critical to successful design. The increasing focus in education on specific fields of engineering science has created a disconnect between the necessary and provided skill set of graduating engineers. Project-based learning (PBL) embodies a number of principles that attempt to help overcome this mismatch.

Practice-oriented learning processes positively influence the outcome of learning [17]. In particular, cooperative education and industry internships increase self-efficacy of students as defined by the perceived level of task competency of an individual. It is found that the quality of instruction in the classroom must reflect the experience of the student during the co-op [17]. Otherwise the applied engineering experience of a teacher is questioned. To prevent alienation resulting from the lack of opportunity for returning students to demonstrate their new knowledge in class, PBL is an avenue to transform the teaching style with positive effects to both sides.

As such, the presented approach supports the reliance of PBL on problem authenticity. While information technology

advances in Model-Based Design have been shown valuable for teaching purposes (e.g., model coverage [18] and code generation [19]), in this paper a virtualization of a medium-sized Cyber-Physical System (CPS) is presented. The virtual use case maintains sufficient detail to enable exploration of authentic CPS issues including direction to multi-domain complications.

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