Algorithms and Numerical Methods for Motion Planning and Motion Control: Dynamic Manipulation Assignments

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Outline

Non-prehensile manipulation assignments
  Examples and features
  A “Butterfly” robot example

Steps in Motion Planning
  Choices of Coordinates
  Searching a Motion and a Motion Generator

Steps in Synthesis of Controller
  Transverse Dynamics and Transverse Coordinates

Concluding remarks
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Concluding remarks
Non-prehensile manipulation: examples and features

Non-prehensile manipulation is manipulation of an object without a form or force-closure grasp
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Dynamic non-prehensile manipulation offers:

- New robot primitives
- Flexibility in grasping objects of different shape and size
- Increased volume of reachable states in “robot+object” workspace
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The challenges in developing algorithms are
- the need in developing dynamical models for a robot, an object and their interaction;
- under-actuation, i.e. when a number of degrees of freedom is larger than a number of actuators;
- the presence of unilateral constraints
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Conceptual scheme of a “Butterfly” robot

The tasks are

- to plan a rolling of a sphere on a frame
- to design a feedback controller to stabilize a motion

One of open challenges in robotics for two decades !!!
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Choices of coordinates

Each body in plane requires 3 coordinates \((x, y, \theta)\) for describing its configuration.

The hand has a fixed point, therefore 4 quantities will be enough for reconstruction of the status of the hand and the disc.
Choices of coordinates

- Detecting of a contact point between the hand and the disc is challenging
- Contact models of rolling are simplistic
Choices of coordinates

- $\theta$ is the angle of rotation of the hand
- $s$ is the distance to go to the shortest to the center of the disc point along the virtual curve $\gamma_c$
- $w$ is the distance to that point along the normal direction
- $\psi$ is the angle of rotation of the disc in the hand frame
Properties valid for rolling without slipping

- \( w \) is zero, i.e. the hand and the disc has a point contact

- \( \frac{d}{dt}s = -R \frac{d}{dt}\psi \), i.e. the disc does not slip when rolls
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Dynamics of the system

The dynamic model of the system in excessive coordinates \( q = (\theta, \psi, s, w) \) is

\[
M_e(q)\ddot{q} + C_e(q, \dot{q})\dot{q} + G_e(q) = [u, 0, 0, 0]^T + F_1 + F_2
\]

with \( u \) being a control variable; \( F_1, F_2 \) being the corresponding reaction forces.
Dynamics of the system

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with $u$ being a control variable; $F_1, F_2$ being the corresponding reaction forces.

The dynamics admit the reduction of the model for two variables $(\theta, \psi)$

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} u \\ 0 \end{bmatrix}$$
Dynamics of the system

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\end{bmatrix}
\begin{bmatrix}
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  \ddot{\psi}
\end{bmatrix}
+ \begin{bmatrix}
  c_{11} & c_{12} \\
  c_{21} & c_{22}
\end{bmatrix}
\begin{bmatrix}
  \dot{\theta} \\
  \dot{\psi}
\end{bmatrix}
+ \begin{bmatrix}
  g_1 \\
  g_2
\end{bmatrix}
= \begin{bmatrix}
  u \\
  0
\end{bmatrix}
\]

We have to search for a forced periodic solution

\[
\theta^*(t), \quad \psi^*(t), \quad u^*(t)
\]

of this system, for which the constraints hold.
if the motion $[\theta^*(t), \psi^*(t)]$ is found then the disc will roll in one direction

\[ \downarrow \]

the angle $\psi^*(t)$ can be used for representation instead of time: $\theta^*(t) = \Phi(\psi^*(t))$
Motion Generator

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the angle \(\psi^*(t)\) can be used for representation instead of time: \(\theta^*(t) = \Phi(\psi^*(t))\)

\[\downarrow\]

\[
\frac{d}{dt} \theta^* = \frac{d}{d\psi} \Phi \frac{d}{dt} \psi^*, \quad \frac{d^2}{dt^2} \theta^* = \frac{d}{d\psi} \Phi \frac{d^2}{dt^2} \psi^* + \frac{d^2}{d\psi^2} \Phi \left[ \frac{d}{dt} \psi^* \right]^2
\]
if the motion \([\theta^*(t), \psi^*(t)]\) is found then the disc will roll in one direction

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the angle \(\psi^*(t)\) can be used for representation instead of time: \(\theta^*(t) = \Phi(\psi^*(t))\)

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\]

\[
\downarrow
\]

The passive dynamics for that motion

\[
m_{21} \ddot{\theta} + m_{22} \ddot{\psi} + c_{21} \dot{\theta} + c_{22} \dot{\psi} + g_2 = 0
\]

can be re-written as a differential equation – Motion Generator – for \(\psi\)-variable

\[
m_{21} \left[ \Phi' \ddot{\psi} + \Phi'' \dot{\psi}^2 \right] + m_{22} \ddot{\psi} + c_{21} \Phi' \dot{\psi} + c_{22} \dot{\psi} + g_2 = 0
\]

\[
\left[\alpha(\psi, \{k_i\}) \ddot{\psi} + \beta(\psi, \{k_i\}) \dot{\psi}^2 + \gamma(\psi, \{k_i\}) = 0\right]
\]
Motion Planning Algorithm

- Choose a set of synchronization functions: $\theta = \Phi(\psi, k_1, \ldots, k_n)$

- Compute a family of Motion Generators parametrized by $k_1, \ldots, k_n$

- Search for constants $k_1^*, \ldots, k_n^*$ and one of the MG solution $\psi^*(t)$ such that the pair

  \[ \psi^*(t), \quad \theta^*(t) = \Phi(\psi^*(t), k_1^*, \ldots, k_n^*) \]

  meets the unilateral constraint

- If so, compute control variable $u^*(t)$ from the system dynamics

  \[ m_{11} \ddot{\theta}^* + m_{12} \ddot{\psi}^* + c_{11} \dot{\theta}^* + c_{12} \dot{\psi}^* + g_1 = u \]

  Otherwise, modify the parameters and re-do the search.
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Analysis of (in)stability of the nominal periodic trajectory and its stabilization are defined by properties of the dynamics transverse to the orbit.

The system state vector has four components \([\theta, \psi, \dot{\theta}, \dot{\psi}]\) \(\Rightarrow\) the dimension of the transverse dynamics is three.

We need to define these quantities for \([\theta^*, \psi^*]\) and regulate them to zero.
Important Remarks

- Non-prehensile manipulation tasks for robotics require combination of:
  - symbolic tools for modeling the dynamics and contact conditions
  - identification and verification procedures for adjusting model parameters
  - optimization procedures for model based trajectory planning
  - control design methods and their stable numerical realization for orbital stabilization

- Solving the task allows students practicing in integration of several advanced computational tools for approaching one of challenging real-world problem in the lab

- Solving the task allows students learning advantages and weaknesses of analytical arguments, numerical procedures and their complementarity in applications

- Safety, safety and safety versus entertainment and challenge in student labs