Optimieren mit MATLAB – jetzt auch gemischt-ganzzahlig

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Let’s consider the following modeling case study…

<table>
<thead>
<tr>
<th>Item</th>
<th>Nuts</th>
<th>Bolts</th>
<th>Revenue per Item</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gadget</td>
<td>5</td>
<td>2</td>
<td>$3.00</td>
</tr>
<tr>
<td>Widget</td>
<td>3</td>
<td>8</td>
<td>$10.00</td>
</tr>
</tbody>
</table>

Current Inventory

<table>
<thead>
<tr>
<th></th>
<th>Nuts</th>
<th>Bolts</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>34</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

How many Gadget / Widget to produce to maximize revenue
How many / at maximum revenue?

- A non-integer solution
  - 3.8235
  - 3.2941
  - Could attempt to round up/down

- The integer solution

$\begin{align*}
\text{not possible} & \rightarrow \$ 44.11 \\
\rightarrow \$ 42.00 & \rightarrow \\
\rightarrow \$ 43.00 & \rightarrow
\end{align*}$
Applications of Optimization

For example

Portfolio Management

Power Generation

- Maximize profits
- Minimize costs
- Maximize efficiency
- Minimize risk

Motor Calibration

Manufacturing / Supply Chain
Optimization Problem
In Literature and in MATLAB

Objective Function

\[ \min_{x} f(x) \]

Typically a linear or nonlinear function

Decision variables (can be discrete or integer)

Subject to Constraints

Linear constraints
- inequalities
- equalities
- bounds

\[ Ax \leq b \]
\[ A_{eq}x = b_{eq} \]
\[ l \leq x \leq u \]

Nonlinear constraints
- inequalities
- equalities

\[ c(x) \leq 0 \]
\[ c_{eq}(x) = 0 \]
Approaches in MATLAB

- **Local Optimization**  
  - Finds local minima/maxima  
  - Needs supplying gradients  
  - Applicable for large scale problems with smooth objective function  
  - Faster/fewer function evaluations

- **Global Optimization**  
  - In general, no gradient information required  
  - Solve problems with non-smooth, stochastic, discontinuous objective function
### Some Types of Optimization Problems

<table>
<thead>
<tr>
<th>Constraint Type</th>
<th>Objective Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>Quadratic</td>
</tr>
<tr>
<td>Linear and/or Bounds</td>
<td>Linear Programm (LP)</td>
</tr>
</tbody>
</table>

**When discrete or integer values involved**

<table>
<thead>
<tr>
<th>Constraint Type</th>
<th>Objective Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>Linear Mixed-Integer Programm (MILP)</td>
</tr>
</tbody>
</table>

* - nonlinear minimization
Local Solvers in MATLAB

- Linear or quadratic programming problems
  - \texttt{linprog}: LP problems
  - \texttt{intlinprog}: MILP problems
  - \texttt{quadprog}: QP problems

- Least squares problems
  - \texttt{lsqlin}: Linear LSQs subject to linear/bound constraints
  - \texttt{lsqnonlin}: Nonlinear LSQs subject to bound constraints

- Nonlinear minimization
  - \texttt{fminunc}: Unconstrained
  - \texttt{fmincon}: Linear and nonlinear constraints
    - Supply Gradient and Hessian functions to speed up
Global Solvers in MATLAB

- **Smooth objective and constraints**
  - GlobalSearch, MultiStart: Multiple local solutions
  - GlobalSearch, MultiStart, patternsearch, ga, simulannealbnd: Single global solution

- **Nonsmooth objective or constraints**
  - patternsearch, ga, simulannealbnd: Does not rely on gradient calculation

- **Discrete or integer values**
  - ga: Mixed-Integer Nonlinear Problems (MINLP)
    - Can have any objective function, bounds, and inequality constraints
    - Can indirectly include equality constraints

*Global Optimization Toolbox*

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**Table for Choosing a Solver**
There are six Global Optimization Toolbox solvers:

- **ga** (Genetic Algorithm)
- GlobalSearch
- MultiStart
- patternsearch, also called direct search
- simulannealbnd (Simulated Annealing)
- gamultiobj, which is not a minimizer; see Multiobjective Optimization
Optimization APP

![Optimization Tool Interface](image-url)
Optimization APP
Optimization APP

Generate Code...

% This is an auto generated MATLAB file from Optimization Tool.

%% Start with the default options
options = optimoptions('fmincon');

%% Modify options setting
options = optimoptions(options,'Display', 'off');
options = optimoptions(options,'PlotFcns', { @optimplotx @optimplotfval });
options = optimoptions(options,'Algorithm', 'active-set');

[x,fval,exitflag,output,lambda,grad,hessian] = fmincon(@rosenbrock,x0,[],[],[],[],[],[],[],options);
Speed up Optimization using Built-in Parallel Support

Iteration n:  
1. Evaluate objective function at $X_n = [x_n,y_n]$  
2. Calculate gradient by finite difference approximation  
   $$f'(x_n) = \frac{f(x_n + \Delta x_n) - f(x_n)}{\Delta x_n}$$  
3. Move to  
   $$x_{n+1} = x_n + \alpha \ f'(x_n)$$  

$\rightarrow$ Can run in parallel!
Speed up Optimization using Built-in Parallel Support

1) Gradient Estimation
   - `fmincon`
   - `fminimax`
   - `fgoalattain`

2) Iterative sampling of local solution space
   - `MultiStart`
   - `ga,gamultiobj`
   - `patternsearch`
Key takeaways

1. Solve wide variety of problems
   • Linear, linear mixed-integer, quadratic, nonlinear, least squares
   • Nonlinear, nonsmooth, stochastic, nonlinear mixed-integer

2. MATLAB environment
   • User-friendly graphical interfaces – Apps – with automatic code generation
     • Optimization App
   • Integrated Numeric, Graphics, Symbolic Math
   • Parallel computing

3. Deploying Applications with MATLAB
   • Share applications with end users who do not need MATLAB
     ▪ Stand-alone executables (.exe)
     ▪ Shared libraries (.dll)
     ▪ Software components (.jar, .dll, .com)
Optimization Solvers in MATLAB

Linear
- linprog
- intlinprog

Quadratic
- quadprog

Nonlinear
- fmincon
- multistart
- globalsearch
- patternsearch

Least Squares
- lsqlin
- lsqnonneg
- lsqcurvefit
- lsqnonlin

Nonsmooth or Noisy
- patternsearch
- ga
- simulannealbnd

Multiobjective
- gamultiobj
Backup
Deploy your optimization application

1.) Define the user interface
2.) Package the application using MATLAB Compiler
3.) Give the application installer to someone

They will install the application ... and run it on their desktop