Adaptive power control algorithm in cognitive radio based on game theory

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Abstract: Cognitive radio (CR) has achieved increasing attention to improve the spectrum utilisation by allowing the coexistence of primary users (PUs) and cognitive users (CUs) in the same frequency band. As the spectrum of interest is licenced to primary network, power control must be carried out within the CR system so that no excessive interference is caused to PUs. In this study, the problem of power control is investigated in CR systems based on game theory subject to interference power constraint at PU and the signal-to-interference-plus-noise ratio (SINR) constraint of each CU. The objective is to reduce the power consumption caused by some CUs’ SINR over the target value. First, a non-cooperation power control game is formulated, and then an adaptive power control algorithm based on feedback function is proposed. Moreover, the existence and uniqueness of Nash equilibrium are proved. Simulation results show that, compared with other distributed algorithms, the proposed algorithm can reduce the power consumption and overcome the near-far effect.

1 Introduction

Cognitive radio (CR), as a promising technology to enhancing the utilisation efficiency of the scarce radio spectrum, has attracted tremendous interests recently. A key feature of CR technology is to allow cognitive user (CU) to share the same frequency band licenced to primary users (PUs) as long as the secondary transmission does not adversely affect any PU [1]. As a result, the primary goal of the CR network is to protect the PUs from harmful interference induced by the CUs as well as to meet the quality of service (QoS) demands of CUs. This transmission strategy is termed as spectrum sharing [2].

As an effective interference suppression technology, power control has extensively been studied in CR [3, 4]. Different from the traditional communication systems, in CR network, the interference constraints of PUs is the first-line issue that CUs should consider. Zhang et al. [5] has focused on the design of CR network subject to the given interference power constraints for protecting the PUs. Zhang et al. have also discussed some promising rules on how to optimally set the interference temperature constraints in the CR network to achieve the best spectrum sharing throughput. In [6], the optimal power allocation strategies to achieve the ergodic capacity and the outage capacity of the CU fading channel were studied, subject to different combinations of peak/average transmit and/or peak/average interference power constraints. A robust power control scheme via link gain pricing with error estimator for cognitive spectrum underlay network was proposed in [7]. The scheme guarantee that the interference temperature of the PUs through operating in the network-centric manner, and keep the fairness between the CUs through link gain pricing. Recently, orthogonal frequency division multiplexing (OFDM) is recognised as an attractive modulation technique for underlay spectrum sharing scenario because of its spectrum shaping flexibility and adaptivity in allocating vacant radio resources. Xu et al. [8] investigated joint subcarrier and bit optimisation in OFDM-based CR networks via PUs’ cooperation under the constraint of CU transmission rate and signal-to-interference-plus-noise ratio (SINR) at PU receiver, where CUs opportunistically utilise the subcarriers through power control. In [9], a multi-objective optimisation approach was proposed to jointly maximise the CU throughput and minimise its transmit power in OFDM-based CR systems, subject to total transmit power threshold and predefined co-channel interference and adjacent channel interference constraints to existing PUs. Furthermore, the more practical assumption of only knowing the path loss on the links between the secondary user transmitter and the PUs receivers were considered.

Game theory is a mathematical tool that is very suitable for resolving conflicts among decision makers in decentralised and self-organised networks [10]. Thanks to multiple non-cooperative CUs sharing the same frequency band licenced to a PU, game theory can be naturally applied to CR networks to solve interactions among CUs. A typical solution of these non-cooperative games is Nash equilibrium (NE). Non-cooperation power control game (NPG-LP) algorithm based on payoff function was proposed [13], which can reduce transmission power mostly by choosing SINR level appropriately. Although the algorithm can improve the CUs’ utility greatly, it did not consider the SINR constraint of each CU and the interference power constraints at PUs. Sun and Zhang [14] further considered both power and rate control using a game-theoretic approach, where multiple CUs are considered. In [15], Li et al. considered not only the QoS requirement as other game algorithms but also the influence of power threshold by introducing the relevant interference model. According to different power thresholds, an approach is proposed to solve the problem of coexistence between PU and CU in CR. Joint spectrum management and power allocation in multiple-input–multiple-output cognitive networks was studied in [16] but the problem was non-convex. To solve this problem, the authors translated it into a non-cooperative game. They then derived an optimal pricing policy for each CU, which adapts to the CU’s neighbouring conditions and drives the game to an NE.

Comparing with the previous work on power control in CR, in this paper, we study the power control problem in cognitive uplink under a game-theoretic framework. The objective is to reduce power waste caused by some CUs’ SINR over the target value. A non-cooperative game is used to analyse this situation and the NE is considered as the...
solution of this game. Simulation results show that the proposed algorithm can reduce the power consumption and overcome the near-far effect under interference power constraint at PU and the SINR constraint of each CU.

The rest of this paper is organised as follows. In Section 2, the system model and problem formulation is presented. In Section 3, a non-cooperative power control game is formulated and an adaptive power control algorithm is proposed. Moreover, the existence and uniqueness of NE is proved. Numerical results are presented in Section 4. Concluding remarks are made in Section 5.

2 System model and problem formulation

We consider the uplink of CR network which shares the spectrum resource with a primary network, as shown in Fig. 1. The primary network consists of a primary base station and a single PU. The secondary network consists of a single cognitive base station (CBS) and N CUs. For the sake of simplicity, all the base stations and users are equipped with only one antenna. Let $h_i$, $i \in \{1, N\}$ denote the channel gain from the CBS to the $i$th CU, $h_0$ denotes the channel gain from PU to CBS. Similarly, channel between the $i$th CU and the PU is denoted by $g_i$, $i \in \{1, N\}$.

The SINR of the $i$th CU is calculated by [15]

$$\gamma_i = \frac{p_i h_i}{\sum_{j=1, j \neq i}^{N} p_j h_j + p_0 h_0 + \sigma^2}$$

(1)

where $p_i$ and $p_0$ are the transmission power of the $i$th CU and PU, respectively. Moreover, background noise is an additive white Gaussian noise with zero mean and variance $\sigma^2$.

To protect the communications of the PU, the peak interference power from CUs should not exceed a predefined threshold $I_{th}$ at the PU. Therefore in order to satisfy the peak interference power constraint, the total received power at a specified PU must meet [17]

$$\sum_{i=1}^{N} p_i g_i \leq I_{th}$$

(2)

In the CR network, to guarantee the QoS requirements of CUs, the received SINR of each CU should be greater than a threshold

$$\gamma_i \geq \gamma_i^b$$

(3)

where $\gamma_i^b$ is the target SINR at CU $i$ to ensure QoS requirement of CU $i$.

3 Non-cooperative power game

In the following sections, we choose an appropriate utility function and obtain the corresponding power iterative algorithm. Then, we prove the existence and uniqueness of the NE.

3.1 Utility function

Game theory is a powerful tool in modelling interactions between self-interested players and predicting their choice of strategies. A utility function of a player quantifies the preference over the game’s possible outcomes. The players in a game with conflict interests will selfishly choose their own strategies to maximise their utility functions. In CR system, if a CU selfishly raises transmission power to increase its own utility, this will inevitably increase the interference to other CUs, thus resulting in mutual interference between CUs [18]. To solve this problem, a balance point should be found when CUs use channel to transmit data, so the power control problem can be abstracted as a non-cooperative power control game from the perspective of game theory. On the basis of the system model described above, the non-cooperative game can be formulated as follows [19]

$$G = [N, \{P_i\}, \{u_i(\cdot)\}]$$

(4)

It has the following three primary components:

- **Players:** In this paper, players are CUs. A finite set of sensor nodes is denoted as $i = \{1, 2, 3, ..., N\}$.
- **Strategic space:** It is defined by the transmission power allocation strategy. For each available power level $p_i \in s_i$, the strategy space is defined as $S = p_1 \times p_2 \times \cdots \times p_N$.
- **Utility function:** The utility function of CU $i$ is denoted as $U(p_i, p_{-i})$, where $p_{-i}$ is the power vector of all CUs, except CU $i$.

To ensure positivity and convexity of the utility function, here, we should consider the SINR requirement of CU $i$ as $\gamma_i^b$ and the peak transmission power constraint of CU $i$ as $P_i^{\max}$ ($0 \leq p_i \leq P_i^{\max}$) to select utility function properly

$$u_i(p_i, p_{-i}) = a_i \log(\gamma_i - \gamma_i^b) + b_i \sqrt{P_i^{\max} - p_i}$$

(5)

where $a_i$ and $b_i$ are constant non-negative weighting factors. In the utility function equation, the selection of weighting factors $a_i$ and $b_i$ is important. On the basis of the interference degree, we can choose greater power when PU is far away from cognitive network and the minor one on the contrary. In accordance with the service requirement, we should properly choose SINR threshold $\gamma_i^b$. If $\gamma_i < \gamma_i^b$ (here, $\gamma_i^b$ denotes the target SINR of the $i$th CU), $a_i$ and $b_i$ remain unchanged, if $\gamma_i > \gamma_i^b$, we can adjust the parameter $b_i = b_i(\gamma_i/\gamma_i^b)$ adaptively, which reduces the interference to other users by increasing the punishment for itself.

3.2 Power control iterative algorithm

In this section, we present an iterative algorithm that repeats the power control steps until convergence. In an NE point, every player is unilaterally optimal and no player can increase its utility alone by changing its own strategy.

**Definition 1:** (NE): A power vector $S = \{p_1, p_2, ..., p_N\}$ is NE of the game $G$ [11]. For all $i \in N$, if and only if

$$u_i(p_i, p_{-i}) \geq u_j(p_j, p_{-j}), \quad \forall p_j \in s_j$$

(6)

where $p_{-i}$ is the power vector of all CUs, except CU $i$ and $p_i^*$ is the NE solution of CU $i$.
To obtain NE solution, by taking the first derivative of \( u(p_i, p_{-i}) \) with respect to \( p_i \), it can be seen that

\[
\frac{\partial u}{\partial p_i} = \frac{a_i}{\gamma_i - \gamma} \cdot \frac{h_i}{J_{(p_{-i})}} - \frac{b_i}{2\sqrt{p_{i}^{max} - p_i}}
\]

where \( f(p_{-i}) = \sum_{j \neq i} p_j h_j + p_i h_0 + \sigma^2 \) represents the interference items from other CUs, PU and background noise.

Then by setting the first derivatives to zero, we obtain

\[
\gamma_i = \gamma_i^* + \frac{2a_i h_i}{b_i J_{(p_{-i})}} \sqrt{p_{i}^{max} - p_i}
\]

Substituting (1) and isolating \( p_i \), into (8), we derive the \( i \)'th CU’s power optimum point

\[
p_i = \frac{f(p_{-i})}{h_i} \gamma_i^* + \frac{2a_i}{b_i} \sqrt{p_{i}^{max} - p_i}
\]

According to (1) and (9), we can obtain

\[
p_i^{(m+1)} = \frac{p_i^{(m)}}{\gamma_i^*} \gamma_i^* + \frac{2a_i}{b_i} \sqrt{p_{i}^{max} - p_i^{(m)}}
\]

On the basis of (10), the corresponding \( i \)'th CU’s power iteration function is given by

\[
p_i^{(m+1)} = \begin{cases} 
\frac{p_i^{(m)}}{\gamma_i^*} \gamma_i^* + \frac{2a_i}{b_i} \sqrt{p_{i}^{max} - p_i^{(m)}}, & 0 < p_i^{(m+1)} < p_i^{max} \\
p_i^{max}, & p_i^{(m+1)} \geq p_i^{max}
\end{cases}
\]

Each CU updates its action at each iteration such that the utility function in (5) is maximised. The adaptive non-cooperative power control algorithm (ANCPCA) is given in Table 1. In the following ANCPA, according to (11), update the power iteration function \( p_i^{(m+1)} \). If satisfies the power interference constraint in (2), it continues; otherwise, iteration stops. For \( i \)'th CU, compute the difference of utility function for \( m + 1 \) and \( m \) iteration. If the difference in allowable error scope \( \varepsilon \), the iteration stops; otherwise, it returns to step 2.

### 3.3 Existence and uniqueness of NE

To analyse the outcome of the game, the achievement of an NE is a well-known optimality criterion.

**Theorem 1:** (existence): At least one NE exists for the proposed game G if, for all \( i \in N \) [15]:

1. The transmission power \( p_i \) is a non-empty, convex and compact subset of some Euclidean space.
2. The utility function \( u(p_i, p_{-i}) \) is continuous and quasi-concave in \( p_i \).

**Proof:** Obviously, the first condition could be satisfied. From (5), it is seen that \( u(p_i, p_{-i}) \) is continuous. Therefore only the quasi-concave property needs to be proved.

The second derivative of \( u(p_i, p_{-i}) \) with respect to \( p_i \) is described as

\[
\frac{\partial^2 u}{\partial p_i^2} = \frac{a_i}{(\gamma_i - \gamma)^2} - b_i - \frac{1}{4\sqrt{(p_{i}^{max} - p_i)^3}}
\]

It is easy to check that \( \frac{\partial^2 u(p_i, p_{-i})}{\partial p_i^2} \leq 0 \), so \( u(p_i, p_{-i}) \) is concave in \( p_i \). As a consequence, the utility functions of CUs satisfy all the required conditions. This proves that NE exists in the game.

**Theorem 2:** The NE of game G is unique.

**Proof:** The key aspect of the uniqueness of the NE is that the best response function is a standard function. To prove that the NE is unique, the best response function

\[
f(p_i) = \frac{2a_i}{b_i} \sqrt{p_{i}^{max} - p_i} + \frac{p_i^*}{\gamma_i^*}
\]

should be a standard function, which fulfills the following axioms [19]:

1. **Positivity:** For all \( i \in N \), \( f(p_i) > 0 \).
2. **Monotonicity:** If \( p_i > p_i' \), \( f(p_i) > f(p_i') \).
3. **Scalability:** For all \( \alpha > 1 \), \( \alpha f(p_i) > f(\alpha p_i) \).

**Positivity:** To ensure the normal operation of the system, the best response function must meet the positivity condition.

**Monotonicity:** If \( p_i > p_i' \),

\[
f(p_i) - f(p_i') = \left[ \frac{\gamma_i^*}{\gamma_i^*} \left( \sqrt{p_{i}^{max} - p_i} + \sqrt{p_{i}^{max} - p_i'} \right) - \frac{2a_i}{b_i} \right]
\]

Owing to

\[
\sum_{i \in N} p_i g_i \leq p_i^* \Rightarrow p_i < \left( p_i^* - \sum_{i \in N} p_i g_i \right)/g_i
\]

let \( p_i - \sum_{i \in N} p_i g_i / g_i = p_i - \sum_{i \in N} p_i g_i / g_i = \beta \), it can be observed that

\[
\sqrt{p_{i}^{max} - p_i} + \sqrt{p_{i}^{max} - p_i'} > 2\sqrt{p_{i}^{max} - \beta}\]

As \( \sqrt{p_{i}^{max} - p_i} - \sqrt{p_{i}^{max} - p_i'} > 0 \) and \( a_i/b_i < \gamma_i^* / \gamma_i^* \sqrt{p_{i}^{max} - \beta} \), so \( f(p_i) - f(p_i') > 0 \). Apparently, the monotonicity property is satisfied.

**Scalability:** For all \( \alpha > 1 \), we have

\[
of(p_i) - f(\alpha p_i) = \frac{2a_i}{b_i} \left( \alpha \sqrt{p_{i}^{max} - p_i} - \sqrt{p_{i}^{max} - \alpha p_i} \right)
\]

Owing to \( \alpha > 1 \), \( \alpha \sqrt{p_{i}^{max} - p_i} > \sqrt{p_{i}^{max} - \alpha p_i} \), so \( of(p_i) > f(\alpha p_i) \). From the above proof, we can see that the best response function \( f(p_i) \) is a standard function, that is, it has only one solution, which completes the NE uniqueness proof.

### 4 Numerical results

In this section, the performance of the proposed scheme is evaluated by computer simulation. We assume that the channel model of \( i \)'th CU is \( h_i = A_i / d_i^4 \), here, \( A = 0.0075 \) is a constant and \( d = [550, 580, 600, 620, 640, 950, 1000, 1050] \) m is the distance from the
ith CU to CBS, and the channel gain $g_i$ is assumed to be Rayleigh flat fading channel. The parameters setting are as follows: the number of CUs is $N = 8$, the maximum transmission power of the ith CU is limited to $p_i^{\max} = 0.5$ W and the initial transmission power is $p_i(0) = 5 \times 10^{-15}$ W, the PU transmission power is $p_0 = 0.05$ W. The SINR threshold is specified as $\gamma_i^h = 7$ dB, background noise $\sigma^2 = 5 \times 10^{-15}$ W, $e = 10^{-15}$ and $2a_i/b_i = 3 \times 10^{-4}$.

To analyse the convergence performance of our algorithms, we randomly generate eight CUs located in CR cell. As shown in Fig. 2, we find that the proposed algorithm needs only about 15–20 iterations to reach convergence. This means the algorithm can fast converge to a pure strategy NE, and accommodate the dynamic network environment. As the distance from CBS becomes farther, the transmission power increases (the top three curves).

To illustrate the advantages of the proposed algorithm, we compare the SINR effects of the proposed algorithm with other two algorithms.

The Koskie and Gajic’s (K-G) algorithm is a well-known iterative method which is also obtained via game theory. It is shown in the following equation

$$p_i^{(m+1)} = \frac{p_i^{(m)}}{\gamma_i^{(m)}} \gamma_i^h \left( \frac{p_i^{(m)}}{\gamma_i^{(m)}} \right)^2$$  \hspace{1cm} (15)

where $b/a$ is the positive correlation coefficient.

Moreover, the signal-to-interference ratio (SIR) Nash algorithm is also given by

$$p_i^{(m+1)} = \frac{p_i^{(m)}}{\gamma_i^{(m)}} \gamma_i^h$$  \hspace{1cm} (16)

From Figs. 3 a–c, we can see that the SINR value of ANCPCA and SIR Nash algorithm can meet the threshold requirement but the SINR of K-G algorithm is not ideal which part of CUs are below the SINR threshold. Therefore it is unfairness for all CUs because the K-G algorithm cannot guarantee the QoS requirements of each CU. Fig. 3a describes the SINR convergence of the ANCPCA. It apparently converges fast to the maximum utility after 15–20 iterations. As we can see, the SINR of each CU of our proposed algorithm converges almost close to the threshold, which demonstrates the ANCPCA can overcome the near-far effect.

In Fig. 4a, we compare the average powers of three power control algorithms with different CUs. It turns out that the SIR Nash approach consumes the most power because of the CUs’ self-interested non-cooperative behaviour in the game process. For example, when one of the CUs cannot reach or maintain its minimum SINR, it resorts to the only means of increasing its transmission power, as do other CUs in a similar situation. As a result, large mutual interference arises among these selfish CUs. For the ANCPCA approach, on the other hand, CUs can perceive
the interference environment well and accordingly make the most appropriate transmission power-adjustment decision. Overall, the K–G algorithm consumes the least power. As shown in Fig. 4b, with the increasing of CUs, the SINR of ANCPCA and K–G algorithm decreases. When the number of CUs exceeds 30, K–G algorithm cannot meet CUs’ need but the ANCPCA approach still can accommodate CU’s requirement, although they might consume high transmission power. Those results show that the ANCPCA approach not only maintains fairness among multiple CUs (CUs close to CBS enjoy high utility, whereas those CUs far away CBS can obtain the basic communication requirements in terms of SINR threshold) but also guarantees the minimum SINR requirements of all CUs.

5 Conclusion

In this paper, the problem of power control was studied for CR system via game theory and the problem was formulated as a non-cooperative power control game. The objective was to reduce the power consumption caused by some CUs’ SINR over the target value, subject to peak interference power constraint at PU and the SINR requirement of each CU. Moreover, an ANCPCA was proposed. Besides, the existence and uniqueness of NE was proved. Simulation results show that the proposed algorithm can decrease power consumption and overcome the near-far effect. The proposed ANCPCA algorithm can also be extended to CR system consisting of multiple PUs and multiple CUs in future work.

6 Acknowledgments

The authors thank the reviewers for their detailed reviews and constructive comments, which have helped improve the quality of this paper. This research was supported by the National Natural Science Foundation and Civil Aviation Administration of China (61071104).

7 References