Computer-Aided Design and Optimization of High-Efficiency LLC Series Resonant Converter

Ruiyang Yu, Godwin Kwun Yuan Ho, Bryan Man Hay Pong, Senior Member, IEEE, Bingo Wing-Kuen Ling, Senior Member, IEEE, and James Lam, Senior Member, IEEE

Abstract—High conversion efficiency is desired in switch mode power supply converters. Computer-aided design optimization is emerging as a promising way to design power converters. In this work a systematic optimization procedure is proposed to optimize LLC series resonant converter full load efficiency. A mode solver technique is proposed to handle LLC converter steady-state solutions. The mode solver utilizes numerical nonlinear programming techniques to solve LLC-state equations and determine operation mode. Loss models are provided to calculate total component losses using the current and voltage information derived from the mode solver. The calculated efficiency serves as the objective function to optimize the converter efficiency. A prototype 300-W 400-V to 12-V LLC converter is built using the optimization results. Details of design variables, boundaries, equality/inequality constraints, and loss distributions are given. An experimental full-load efficiency of 97.07% is achieved compared to a calculated 97.4% efficiency. The proposed optimization procedure is an effective way to design high-efficiency LLC converters.

Index Terms—Computer-aided design, efficiency, LLC resonant converter, optimization, power converter.

NOMENCLATURE

\( a, b, c \) Curve fitting factor.
\( a_{DF}, b_{DF} \) Curve fitting factor.
\( A_r \) Effective cross-sectional area of transformer.
\( A_{r,Lr} \) Effective cross-sectional area of resonant inductor.
\( b_{xl} \) Lower bound vector of design variables.
\( b_{yu} \) Upper bound vector of design variables.
\( B_{m,XF} \) Peak-to-peak swing of transformer flux density.
\( \Delta B_{m,XF} \) Amplitude of transformer flux density swing.
\( \Delta B_{m,Lr} \) Amplitude of resonant inductor flux density swing.
\( C_r \) Value of resonant capacitor.

\( d_{AWG} \) Diameter of AWG wire in transformer primary winding.
\( d_{Lr,AWG} \) Diameter of AWG wire in resonant inductor winding.
\( D_F \) Dissipation factor.
\( f \) Turn-off energy consumed by primary MOSFET.
\( f_c \) Frequency.
\( f_r \) Resonant frequency \((L_r C_r)\).
\( f_s \) Switching frequency.
\( F \) Normalized frequency.
\( F_R \) Ratio of AC–DC resistance.
\( F_{R, pri} \) Transformer primary side \( F_R \).
\( F_{R, sec} \) Transformer secondary side \( F_R \).
\( F_{R, off} \) Resonant inductor \( F_{R, on} \).
\( h_{foil} \) Thickness of foils in transformer secondary winding.
\( I_{Lr} \) Resonant inductor current.
\( I_{base} \) Base current for normalization.
\( I_{n,pri} \) \( n \)th harmonic component of primary RMS current.
\( I_{n,sec} \) \( n \)th harmonic component of secondary RMS current.
\( I_{LR, MAX} \) Maximum resonant inductor current.
\( I_{off} \) Turn-off current of primary MOSFET.
\( I_{rip, in} \) Input ripple current.
\( I_{rip, out} \) Output ripple current.
\( I_{RMS, pri} \) Primary side RMS current.
\( I_{RMS, sec} \) Secondary side RMS current.
\( j_{Lr} \) Normalized resonant inductor current.
\( j_{Lm} \) Normalized magnetizing current.
\( j_{out} \) Normalized output current.
\( j_{RMS} \) Normalized secondary rectified current.
\( k \) Steinmetz coefficient.
\( k_1 \) Ratio of two resonant frequencies.
\( k_2 \) Ratio of turn-off energy and turn-off voltage.
\( k_{off} \) Resonant inductor value.
\( k_{off} \) Naganizing inductor value.
\( m_{1}, m_{2} \) Normalized input/output voltage.
\( m_c \) Normalized resonant capacitor voltage.
\( m_{Lr} \) Normalized resonant inductor voltage.
\( m_m \) Normalized transformer voltage.
\( m_{m2} \) Normalized transformer voltage (mode indicator).
\( M \) Normalized conversion ratio.
\( n \) Order of harmonic frequency.
\( n_{C, in} \) Number of input capacitor parallelled.

Manuscript received June 22, 2011; revised August 26, 2011 and October 12, 2011; accepted November 23, 2011. Date of current version April 3, 2012.
Recommended for publication by Associate Editor D. Xu.
R. Yu, G. K. Y. Ho, and B. M. H. Pong are with the Department of Electrical and Electronic Engineering, The University of Hong Kong, Hong Kong (e-mail: yry721@eee.hku.hk; godwinho@hotmail.com; mph@eee.hku.hk).
B. W.-K. Ling is with the School of Engineering, University of Lincoln, Lincolnshire, LN6 7TS, U.K. (e-mail: wling@lincoln.ac.uk).
J. Lam is with the Department of Mechanical Engineering, The University of Hong Kong, Hong Kong (e-mail: james.lam@hku.hk).
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Digital Object Identifier 10.1109/TPEL.2011.2179562
I. INTRODUCTION

Energy efficiency is a hot topic that has drawn the attention of researchers and engineers for decades. Numerous research works have focused on improving power converter efficiency. Computer-aided design optimization is one of the methods used to achieve high-energy conversion efficiency, and it has been applied widely in conventional PWM converter design. Early research work [1] utilized the sequential unconstrained minimization technique (SUMT) or the augmented Lagrangian (ALAG) penalty function technique to optimize the converter mass. A practical converter optimization approach was developed in [2] for industrial applications, which utilized the nonlinear optimization program to optimize converter design. Design optimization of interleaved converter for automobile applications was investigated in [3]. A Monte Carlo searching method was applied to handle a large number of design variables. Fuel cell system mass was minimized in [4] under a certain duration constraint. The Pareto-front of power converter multiobject optimization was investigated by [5]. The Pareto-front of converter volume and efficiency were obtained, which means no further efficiency improvement can be achieved under certain constraints, such as converter volume or mass. Converter volume and efficiency were included in the weighted objective function to determine the degree of optimized efficiency or volume. The Pareto-front curve of power density versus efficiency showed that the optimized efficiency was limited by a certain volume constraint. A similar optimization approach was applied to phase-shift PWM converter design in [6] to achieve 99% efficiency.

LLC series resonant converter is emerging to meet the high-efficiency requirements of offline converter and it is becoming increasingly popular in industrial applications. It has zero-voltage switching (ZVS) at primary side and zero-current switching (ZCS) at secondary side. Design methodology of 1-MHz 1-kW LLC converter was investigated in [7]. Details of design procedure were presented in a digital control LLC converter [8]. The LLC converter efficiency can be further improved by using synchronous rectifiers [9]–[11]. Actually, it is more sensible to design and optimize the LLC converter and synchronous rectifiers as an entire system. Adaptive control methodology was proposed to improve the performance of LLC converter [12]. The application of LLC converter in photovoltaic (PV) system was developed in [13]. The advantage of high efficiency from light load to full load shows performance improvements of the entire PV system. Design procedures for wide range LLC converter and dead-time of LLC converter were presented in [14], [15], respectively.
The proposed LLC mode solver serves as a function block in the main optimization procedure. The input variables of the LLC steady-state solver are the values of resonant parameters, such as $L_r$, $C_r$, and $L_m$ and the excitations, such as the switching frequency, load, and input/output voltage. The state equations are solved numerically and the output of this function block are vectors containing particular waveform information of current and voltage. The LLC converter has several modes of operation. These modes include the continuous conduction mode below or above resonance, discontinuous conduction mode below or above resonance, and cut-off mode. Continuous conduction mode is defined as a state in which the secondary diode conducts throughout the switching cycle. Discontinuous conduction mode is defined as the state in which secondary diode has certain periods not conducting. The mode solver presented can tackle different modes, which are determined by the nonlinear relationship of the switching frequency, load, and input/output voltage. The detailed procedures of the LLC mode solver are in Fig. 2.

### B. Normalization

The solver procedures start with normalization, as shown in Fig. 2 (a1) and (a2). The resonant characteristics of the tank circuit are normalized for the sake of uniformity. We use $\omega_0$ and $\omega_1$ to denote the two resonant frequencies

$$\omega_0 = \frac{1}{\sqrt{L_r C_r}}, \quad \omega_1 = \frac{1}{\sqrt{(L_r + L_m)C_r}}. \quad (1)$$

The operation angle $\theta$ is given by

$$\theta = \omega_0 t. \quad (2)$$

Denote $F$ the ratio of two frequencies

$$F = \frac{f_s}{f_r}. \quad (3)$$

A half period of switching cycle $\gamma$ is defined by

$$\gamma = \omega_0 \frac{\theta}{2f_s} = \frac{\pi}{F}. \quad (4)$$

The conversion ratio $M$ is defined as

$$M = \frac{V_2}{V_1}. \quad (5)$$

We define some normalized parameters in the following:

$$V_{\text{base}} = V_2 = \frac{n_p}{n_s} V_{\text{out}}, \quad m_2 = \frac{V_2}{V_{\text{base}}} = 1 \quad (6)$$

$$\begin{align*}
   m_1 &= \frac{1}{M} \quad (7) \\
   Z_{\text{base}} &= \frac{L_r}{C_r} \quad (8) \\
   I_{\text{base}} &= \frac{V_{\text{base}}}{Z_{\text{base}}} \quad (9)
\end{align*}$$

where $V_{\text{base}}$ is defined as the $V_2$ so that $m_2$ is normalized to unity, and $m_1$ is the normalized input voltage. The base impedance $Z_{\text{base}}$ and base current $I_{\text{base}}$ are given by (8) and (9), respectively.
The normalized voltage on resonant capacitor $m_c(\theta)$ and normalized current through resonant inductor $j_{Lr}(\theta)$ are, respectively, given by

$$m_c(\theta) = \frac{v_c(\theta)}{V_{\text{base}}},$$

$$j_{Lr}(\theta) = \frac{i_{Lr}(\theta)}{I_{\text{base}}}. \quad \text{(11)}$$

Similar expressions are applied to $m_m(\theta)$, $m_{m2}(\theta)$, $m_{Lr}(\theta)$, $J_{LM}(\theta)$, and $J_{out}$.

The ratio of two resonant inductance $\lambda$ and the ratio of two resonant frequencies $k_1$ are, respectively, given by

$$\lambda = \frac{L_r}{L_m} = \frac{m_{Lr}(\theta)}{m_m(\theta)} \quad \text{(12)}$$

$$k_1 = \frac{\omega_1}{\omega_0}. \quad \text{(13)}$$

The normalized output load resistance $r_L$ is defined as

$$r_L = \frac{n_L^2 R_L}{n_c^2 Z_{\text{base}}}. \quad \text{(14)}$$

### C. Operation Below Resonant Frequency

1) **Discontinuous Conduction Mode Below Resonance:** If $F < 1$, the LLC is assumed to operate in discontinuous conduction mode below resonance (DCMB) first, as shown in Fig. 2 (a1). DCMB is one of the popular designed operation modes. In DCMB mode, the LLC converter voltage conversion ratio $M$ is larger than unity ($M > 1$). Typical waveforms in DCMB mode are shown in Fig. 3(b). The equivalent circuit of DCMB mode in $\theta \in [0, \alpha)$ is shown in Fig. 4(b). The dead-time transition is ignored for simplified analyses. The state equations are given by (15)

$$\theta \in [0, \alpha)$$

$$\begin{align}
m_c(\theta) &= \left[m_c(0) - \frac{1}{M} + 1\right] \cos(\theta) + j_{Lr}(0) \sin(\theta) \\
m_m(\theta) &= 1 \\
j_{Lr}(\theta) &= \left[-m_c(0) + \frac{1}{M} - 1\right] \sin(\theta) + j_{Lr}(0) \cos(\theta) \\
j_{Lm}(\theta) &= J_{LM}(0) + \lambda \theta.
\end{align} \quad \text{(15a-d)}$$
Fig. 3. LLC operation modes.
The equivalent circuit in $\theta \in [\alpha, \gamma]$ is shown in Fig. 4(c). The state equations are given by (16)

$$
\begin{align}
\begin{cases}
 m_r(\theta) = [m_r(\theta) - \frac{1}{M}] \cos[k_1(\theta - \alpha)] \\
 + \frac{jL_r(\theta)}{k_1} \sin[k_1(\theta - \alpha)] + \frac{1}{M} \\
m_m(\theta) = \{[-m_m(\theta) + \frac{1}{M}] \cos[k_1(\theta - \alpha)] \\
- \frac{jL_r(\theta)}{k_1} \sin[k_1(\theta - \alpha)]\}/(1 + \lambda) \\
 jL_r(\theta) = [-m_m(\theta) + \frac{1}{M}] \sin[k_1(\theta - \alpha)] \\
 + jL_r(\theta) \cos[k_1(\theta - \alpha)] \\
 jL_m(\theta) = jL_r(\theta).
\end{cases}
\end{align}
$$

The average output current $j_{out}$ is given by

$$
\begin{align}
 j_{out} &= \frac{1}{\gamma} \int_{0}^{\gamma} [j_{Lr}(\theta) - j_{Lm}(\theta)] d\theta \\
 &= \frac{1}{\gamma} \int_{0}^{\alpha} [j_{Lr}(\theta) - j_{Lm}(\theta)] d\theta \\
 &= \frac{1}{\gamma} \left\{ \left[-m_{c}(0) + \frac{1}{M} \right](1 - \cos \alpha) \\
+ j_{Lr}(0) \sin \alpha - j_{Lr}(0) \alpha - \frac{1}{2} \lambda \alpha^2 \right\}.
\end{align}
$$

The steady-state solution in DCMB $[j_{Lr}(0), m_c(0), \alpha, M]$ can be solved by

$$
\begin{align}
\begin{cases}
 m_c(0) + m_c(\alpha) = 0 \\
 j_{Lr}(0) + j_{Lr}(\alpha) = 0 \\
 j_{Lr}(\alpha) - j_{Lm}(\alpha) = 0 \\
 j_{out} r_L - 1 = 0.
\end{cases}
\end{align}
$$

These four equations become the basis of the solver, and which adequately describe the waveforms of the resonant operation. The initial condition $m_r(0)$ is equal to $-m_r(\gamma)$, as shown in Fig. 3(b), given by (18a). The same reasoning can be applied to $j_{Lr}(0)$ and $-j_{Lr}(\gamma)$ in (18b). The diode stops conducting at angle $\alpha$, where the resonant current equals to the magnetizing current. Hence, (18c) is formulated that $j_{Lr}(\alpha) = j_{Lm}(\alpha)$. Finally, the unity output voltage is equal to $j_{out} r_L$, given by (18d). These four equations have four unknowns $[j_{Lr}(0), m_c(0), \alpha, M]$. Two unknowns are the normalized boundary value of resonant inductor current $j_{Lr}(0)$ and resonant capacitor voltage $m_r(0)$. The third unknown is the normalized time $\alpha$ that the secondary diode stops conducting. The fourth unknown is the conversion ratio $M$. Since these unknowns do not have the analytical closed-form solution, the equations are solved by MATLAB function \texttt{fsolve}(x), which is a numerical-based search function.

After solving the above four equations, the following procedures are carried out to validate the assumption of DCMB. At the time when $j_{Lr}(\theta) = j_{Lm}(\theta)$ (the moment $\theta = 0$ or interval $\theta \in [\alpha, \gamma]$ in DCMB), the voltage on $L_m$ determines whether the diodes start to conduct or not. A mode indicator $m_{m2}(\theta)$ is defined as the normalized voltage on $L_m$, based on the equivalent circuit Fig. 4(c), when $\theta = 0$ or $\theta \in [\alpha, \gamma]$. According to Kirchhoff’s Voltage Law, we obtain

$$
m_{Lr}(\theta) + m_{m2}(\theta) + m_c(\theta) = m_1. \quad (19)
$$

The solution of $m_{m2}(\theta)$ can be derived by inserting (7) and (12) into (19). To simplify the analyses, we only consider the instants 0 and $\gamma$

$$
m_{m2}(\theta) = \frac{-m_c(\theta) + \frac{1}{2} \lambda}{1 + \lambda} |_{\theta = 0, \gamma}. \quad (20)
$$

$(\theta = 0)$: If $|m_{m2}(0)| \geq 1$ (the output voltage is normalized to 1), the secondary diode conducts and clamps the $m_m(0)$ to 1 (DCMB true).

Otherwise, if $|m_{m2}(0)| < 1$, the secondary diode is OFF and it is no longer DCMB (DCMB false) but in another mode, discontinuous conduction mode above and below resonance (DCMAB), as shown in Fig. 2 (c1) and Fig. 3(c). Since the LLC converter is assumed operating at DCMB at this moment, DCMAB should be considered later.

$(\theta = \gamma)$: At the end of DCMB first half cycle $\gamma$, if $|m_{m2}(\gamma)| \leq 1$, the diode is OFF (DCMB true). Otherwise, if $|m_{m2}(\gamma)| > 1$, the assumption of DCMB is violated (DCMB false).

Summaries are listed as below:

Flow chart Fig. 2 (b1) shows, if $|m_{m2}(0)| \geq 1$ and $|m_{m2}(\gamma)| \leq 1$, the assumption of DCMB is true.

If $|m_{m2}(0)| < 1$ and $|m_{m2}(\gamma)| < 1$, the assumption of DCMB is false then LLC converter is assumed to operate at DCMAB, as shown in Fig. 2 (c1).

If $|m_{m2}(0)| > 1$ and $|m_{m2}(\gamma)| > 1$, the assumption of DCMB is false and then LLC converter is assumed to operate at DCMB2, as shown in Fig. 2 (d1).
D. Operation Above Resonant Frequency

1) Continuous Conduction Mode Above Resonance (CCMA): If the $F > 1$, the LLC is assumed to operate in continuous conduction mode above resonance (CCMA), as shown in Fig. 2 (a2). CCMA is a popular mode in LLC converter operation. In this mode, the LLC converter voltage conversion ratio is less than unity ($M < 1$). Typical waveforms in CCMA mode are shown in Fig. 3(a). The equivalent circuit in CCMA mode in $\theta \in [0, \alpha)$ is the circuit (b) of Fig. 4. The state equations are given by

$$
\begin{align*}
\theta & \in [0, \alpha) \\
m_c(\theta) & = [m_c(0) - \frac{1}{M} + 1] \cos(\theta) + j_{L_r}(0) \sin(\theta) + \frac{1}{M} - 1 \\
m_m(\theta) & = -1 \\
j_{L_r}(\theta) & = [-m_c(0) + \frac{1}{M} + 1] \sin(\theta) + j_{L_r}(0) \cos(\theta) + \frac{1}{M} - 1 \\
j_{L_m}(\theta) & = j_{L_m}(0) - \lambda \theta.
\end{align*}
$$

(22a)–(22d)

The equivalent circuit in $\theta \in [\alpha, \gamma)$ is the circuit (a) in Fig. 4. The state equations are presented as follows:

$$
\begin{align*}
m_c(\theta) & = [m_c(\alpha) - \frac{1}{M} + 1] \cos(\theta - \alpha) + j_{L_r}(\alpha) \sin(\theta - \alpha) + \frac{1}{M} - 1 \\
m_m(\theta) & = 1 \\
j_{L_r}(\theta) & = [-m_c(\alpha) + \frac{1}{M} - 1] \sin(\theta - \alpha) + j_{L_r}(\alpha) \cos(\theta - \alpha) \\
j_{L_m}(\theta) & = j_{L_m}(\alpha) + \lambda \theta.
\end{align*}
$$

(23a)–(23d)

2) Discontinuous Conduction Mode Above or Below Resonance (DCMA and DCMAB): The equivalent circuit in DCMA mode $\theta \in [0, \alpha)$ and $[\beta, \gamma)$ when the LLC converter operates at DCMA, as shown in Fig. 3(c). The equivalent circuit in DCMAB mode $\theta \in [0, \alpha)$ and $[\beta, \gamma)$ is the circuit (c) of Fig. 4. The equivalent circuit in $\theta \in [\alpha, \beta)$ is the circuit (a) of Fig. 4. Five equations are formulated to solve DCMAB given by

$$
\begin{align*}
\theta & \in [\alpha, \gamma) \\
m_c(\theta) & = [m_c(\alpha) - \frac{1}{M} + 1] \cos(\theta - \alpha) + j_{L_r}(\alpha) \sin(\theta - \alpha) + \frac{1}{M} - 1 \\
m_m(\theta) & = 1 \\
j_{L_r}(\theta) & = [-m_c(\alpha) + \frac{1}{M} - 1] \sin(\theta - \alpha) + j_{L_r}(\alpha) \cos(\theta - \alpha) \\
j_{L_m}(\theta) & = j_{L_m}(\alpha) + \lambda \theta.
\end{align*}
$$

(25a)–(25d)

The normalized average output current is given by

$$
\begin{align*}
\frac{1}{\gamma} \int_0^\gamma [j_{L_r}(\theta) - j_{L_m}(\theta)] d\theta \\
& = \frac{1}{\gamma} \int_0^\alpha [j_{L_m}(\theta) - j_{L_r}(\theta)] d\theta \\
& + \frac{1}{\gamma} \int_\alpha^\gamma [j_{L_r}(\theta) - j_{L_m}(\theta)] d\theta \\
& = \frac{1}{\gamma} \left\{ \alpha j_{L_m}(0) - \frac{1}{2} \lambda \alpha^2 - \left[ -m_c(0) + \frac{1}{M} + 1 \right] \right. \\
& \times (1 - \cos \alpha) - j_{L_r}(0) \sin \alpha - \left[ -m_c(\alpha) + \frac{1}{M} - 1 \right] \right.
\\& \times \left[ 1 - \cos(\gamma - \alpha) \right] + j_{L_r}(\alpha) \sin(\gamma - \alpha) \\
& \left. - j_{L_m}((\gamma - \alpha) - \frac{1}{2} \lambda(\gamma - \alpha)^2) \right\}.
\end{align*}
$$

(24)

Four equations are formulated to solve CCMA given by

$$
\begin{align*}
m_c(0) + m_c(\gamma) & = 0 \\
j_{L_r}(0) + j_{L_r}(\gamma) & = 0 \\
j_{L_r}(\alpha) - j_{L_m}(\alpha) & = 0 \\
j_{L_m}(0) - r_L & = 0.
\end{align*}
$$

(25a)–(25d)
TABLE I
OPERATION MODES OF LLC CONVERTER

<table>
<thead>
<tr>
<th>MODE</th>
<th>Angle</th>
<th>Equivalent Circuit</th>
<th>Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCMB</td>
<td>0, α</td>
<td>(b)</td>
<td>$[j_{Lr}(0), m_c(0), \alpha, \delta, M]$</td>
</tr>
<tr>
<td></td>
<td>(α, Γ)</td>
<td>(c)</td>
<td></td>
</tr>
<tr>
<td>DCMAB</td>
<td>0, α</td>
<td>(c)</td>
<td>$[j_{Lr}(0), m_c(0), \alpha, \beta, M]$</td>
</tr>
<tr>
<td></td>
<td>(α, β)</td>
<td>(b)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(β, γ)</td>
<td>(c)</td>
<td></td>
</tr>
<tr>
<td>DCMB2</td>
<td>0, α</td>
<td>(b)</td>
<td>$[j_{Lr}(0), m_c(0), \alpha, \beta, M]$</td>
</tr>
<tr>
<td></td>
<td>(α, β)</td>
<td>(c)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(β, γ)</td>
<td>(a)</td>
<td></td>
</tr>
<tr>
<td>CCMB</td>
<td>0, α</td>
<td>(b)</td>
<td>$[j_{Lr}(0), m_c(0), \alpha, M]$</td>
</tr>
<tr>
<td></td>
<td>(α, γ)</td>
<td>(a)</td>
<td></td>
</tr>
</tbody>
</table>

TABLE II
VALIDATIONS OF LLC OPERATION MODES

<table>
<thead>
<tr>
<th>$F \leq 1$</th>
<th>$m_{m2}(0) \geq 1$</th>
<th>$m_{m2}(0) \leq 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>m_{m2}(γ)</td>
<td>m_{m2}(α) \leq 1</td>
<td>DCMB true</td>
</tr>
<tr>
<td>m_{m2}(γ) \leq 1</td>
<td>DCMB true</td>
<td>N.A.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$F &gt; 1$</th>
<th>$m_{m2}(0) \geq 1$</th>
<th>$m_{m2}(0) \leq 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>m_{m2}(γ)</td>
<td>m_{m2}(α) \leq 1</td>
<td>DCMA true</td>
</tr>
<tr>
<td>m_{m2}(γ) \leq 1</td>
<td>DCMB true</td>
<td>N.A.</td>
</tr>
</tbody>
</table>

Fig. 5. Transformer current waveform and harmonic components.

Fig. 6. PSPICE-simulated primary MOSFET turn-off loss.

A. Primary MOSFET

The most promising feature of the LLC converter is zero voltage switching turn-on and small turn-off current for primary side MOSFETs. A simple and effective MOSFET switching loss model is proposed for the prediction of turn-off switching loss at different turn-off currents, as shown in Fig. 6(a). This proposed model utilizes a curve-fitting method to record SPICE simulation results of turn-off switching loss $E_{off}(I_{off})$. The input voltage is fixed at 330, 365, and 400 V in SPICE simulation, as shown in Fig. 6(a). $E_{off}$ is nearly linearly increasing with $V_{in}$ from 330 to 400 V at a certain turn-off current level, as shown in Fig. 6(b). The parameter $k_{off}$ is defined as the ratio of $E_{off}$ increasing value from 330 to 400 V divided by the voltage increasing value 70 V. The actual energy dissipated during switching is $E_{off}(I_{off}, V_{in})$

$$E_{off}(I_{off}) = ae^{bI_{off}} + c$$  \hspace{1cm} (26)

$$k_{off}(I_{off}) = \frac{E_{off}(I_{off}, 400 \text{ V}) - E_{off}(I_{off}, 330 \text{ V})}{400 \text{ V} - 330 \text{ V}}$$  \hspace{1cm} (27)

$$E_{off}(I_{off}, V_{in}) = E_{off}(I_{off}, 330 \text{ V}) + k_{off}(I_{off})(V_{in} - 330)$$  \hspace{1cm} (28)

It should be noted that during turn-off, there are two currents flowing through the MOSFET and the total energy value is
\[ E_{\text{off}}(I_{\text{off}}, V_{\text{in}}) \]

One current is to charge the output capacitance of MOSFET to \( V_{\text{in}} \) with the energy \( E_{\text{off}}(0, V_{\text{in}}) \), the other current produces energy dissipation (cannot be recovered) in the MOSFET channel with the energy \( E_{\text{off}}(I_{\text{off}}, V_{\text{in}}) - E_{\text{off}}(0, V_{\text{in}}) \). During the dead-time, the energy stored in the output capacitance of MOSFET \( E_{\text{off}}(0, V_{\text{in}}) \) is recovered to the input capacitor (the drain source voltage of MOSFET drops from \( V_{\text{in}} \) to 0, soft switching achieved).

The switching loss and conduction loss of the high side and the low side primary MOSFETs (assuming the same type of MOSFETs at the high side and the low side) are denoted as \( P_{\text{sw, pri}}(I_{\text{off}}, V_{\text{in}}) \) and \( P_{\text{cd, pri}} \), respectively. The gate driving loss of primary MOSFETs is denoted as \( P_{\text{g, pri}} \).

\[
P_{\text{sw, pri}}(I_{\text{off}}, V_{\text{in}}) = 2f_s\left[E_{\text{off}}(I_{\text{off}}, V_{\text{in}}) - E_{\text{off}}(0, V_{\text{in}})\right]
\]

(29)

\[
P_{\text{cd, pri}} = I_{\text{RMS, pri}}^2 R_{d, \text{pri}}
\]

(30)

\[
P_{\text{g, pri}} = Q_{\text{g, pri}} V_{\text{g, pri}} f_s.
\]

(31)

### B. Isolation Transformer

Transformer design of LLC converter is an important task toward achieving high efficiency. Here, sandwich winding is implemented in order to reduce the AC resistance of the transformer. A center tap configuration is applied at secondary with copper foils for high current low voltage applications. Magnetizing inductance is integrated in the isolation transformer with a certain air gap. A typical transformer structure is shown in Fig. 7. The primary and secondary DC resistance \( R_{X, F, \text{pri}} \) and \( R_{X, F, \text{sec}} \) can be directly calculated by the winding geometry. The skin depth of the \( n \)th harmonics frequency is given by

\[
\delta(n) = \frac{2\rho_{\text{cu}}}{2\pi n f_s \mu_0}.
\]

(32)

The AC-to-DC resistance ratio \( R_{R} \) at \( n \)th harmonic frequency is calculated by Dowell’s equation [20, 21], given by

\[
F_R(n, p, X) = \frac{X e^{2X} - e^{-2X} + 2\sin(2X)}{e^{2X} + e^{-2X} - 2\cos(2X)} + \frac{2X^2 - 1}{3} \frac{e^{X} - e^{-X} - 2\sin(X)}{e^{X} + e^{-X} + 2\cos(X)}.
\]

(33)

We have: \( F_R(\text{pri}, n) = F_R(n, p, X) \) is for primary round conductors with \( p = \pi \ell_{\text{layer}} X = \frac{\pi d_{\text{w, pri}}}{2n} \) [21].

\( F_R(\text{sec}, n) = F_R(n, p, X) \) is for secondary foils with \( p = \frac{n}{2} \) and \( X = \frac{d_{\text{w, sec}}}{8(n)} \).

The AC copper loss at each harmonic frequency is calculated by summing the losses from DC to 32nd harmonics. The primary side and secondary copper losses of the transformer are given by

\[
P_{\text{cu, } X, F, \text{pri}} = R_{X, F, \text{pri}} \sum_{n=0}^{32} F_R(\text{pri}, n) I_{n, \text{pri}}^2
\]

(34)

![Fig. 7. Winding structure of transformer.](image-url)

\[
P_{\text{cu, } X, F, \text{sec}} = R_{X, F, \text{sec}} \sum_{n=0}^{32} F_R(\text{sec}, n) I_{n, \text{sec}}^2
\]

(35)

where \( I_{n, \text{pri}} \) and \( I_{n, \text{sec}} \) denote the \( n \)th order harmonic current at the primary side and the secondary side of the transformer. Flux swing of the half-bridge LLC converter is bidirectional. The peak-to-peak flux density is given by

\[
B_{m, X} = \frac{\int_{0}^{\pi/2} |V_{Lm}(t)| dt}{n_p A_{e, X}}.
\]

(36)

The empirical Steinmetz equation [19] is applied to calculate the core loss of the transformer, given by

\[
P_{\text{core, } X} = V_{r, X} f_s k_{\text{core}} \Delta B_{m, X}^2
\]

(37)

where \( \Delta B_{m, X} = \frac{1}{2} B_{m, X} \) is the flux swing, and \( k, \alpha_{\text{core}} \) are the Steinmetz coefficients provided by the manufacturer [22].

### C. Resonant Inductor

A separate resonant inductor is applied in the LLC converter. The separate inductor is used because it simplifies the resonance design process. Integrated transformer may lead to totally different loss models, designs, and optimization procedures.

The losses in the resonant inductor are copper loss and core loss. The DC resistance of resonant inductor is calculated according to its geometry. Dowell’s equation (33) is also applied to calculate AC resistance. \( F_R(\ell, p, X) = F_R(n, p, X) \) is for resonant inductor, with \( p = \ell_{\text{layer}} \) and \( X = \frac{d_{\text{w, pri}}}{2n} \). The copper loss of resonant inductor is given by

\[
P_{\text{cu, } L_{r}} = R_{L_{r}} \sum_{n=0}^{32} F_R(\ell, n) I_{n, \text{pri}}^2
\]

(38)

Core loss of resonant inductor is given by

\[
P_{\text{core, } L_{r}} = V_{r, L_{r}} f_s k_{\text{core}} \Delta B_{m, L_{r}}^2
\]

(39)

where \( \Delta B_{m, L_{r}} = \frac{L_{r}}{n_{L_{r}}} A_{e, L_{r}} \) is the flux swing of resonant inductor.
D. Synchronous Rectifier

Synchronous rectification (SR) is implemented at the secondary side to achieve high efficiency at the low-voltage high-current output condition. The current driven synchronous rectifier driving scheme [23], [24] is implemented. The SR driver is shown in Fig. 8. We assume that the SR works under a timing scheme that current does not flow through synchronous rectifier body diode. The major losses for the synchronous rectifier are the conduction loss, turn-off switching loss, and the gate-drive loss. Turn-off switching loss is the energy stored in the stray inductance and being dissipated by the circuit [25]. The simplified model for the turn-off loss and the gate-driving loss of SR are denoted as $P_{swSR}$ and $P_{gSR}$, respectively. The conduction loss of SRs is denoted as $P_{cdSR}$.

$$P_{swSR} = \frac{n_s V_{in} Q_{ossSR} f_s}{2 n_p}$$  \hspace{1cm} (40)

$$P_{gSR} = Q_{gSR} V_{gSR} f_s$$  \hspace{1cm} (41)

$$P_{cdSR} = I_{RMS,sec}^2 R_{dsSR}.$$  \hspace{1cm} (42)

E. Capacitors

1) Resonant Capacitor: The resonant capacitor in series with the power path carries high RMS current and high voltage. A low-loss capacitor is used to achieve high efficiency and low temperature. A metalized polypropylene capacitor is selected because of its low dissipation factor and low cost. Typically, the dissipation factor (or loss angle $\tan \delta$) of polypropylene capacitor increases with the increasing of frequency up to 10 MHz. Same as before, the curve-fitting method is applied to record the dissipation factor of the resonant capacitor Fig. 9

$$D_F = a_{DF} f + b_{DF}$$  \hspace{1cm} (43)

$$R_{Cr} = \frac{D_F}{2 \pi f C_{Cr}}$$  \hspace{1cm} (44)

$$P_{Cr} = I_{RMS,pri}^2 R_{Cr}$$  \hspace{1cm} (45)

where $D_F$ is the dissipation factor with fitting parameter $a_{DF} = 0.03642$ and $b_{DF} = 2.611$. The equivalent series resistance (ESR) of the resonant capacitor $R_{Cr}$ is calculated according to the dissipation factor and the capacitance, given by (44), and the loss of resonant capacitor $P_{Cr}$ is given by (45).

2) Input/output capacitors: Attention should be paid to the output capacitor selection. The output ripple current of LLC converter is higher than that of PWM converters (such as forward, half-bridge, and Cuk converter). The actual ripple currents are calculated by summing frequency harmonic components (DC component excluded) from fundamental frequency to $32nd$ frequency, given by

$$I_{2rip,in} = \sum_{n=1}^{32} I_{n,prim}^2$$  \hspace{1cm} (46)

$$I_{2rip,sec} = \sum_{n=1}^{32} I_{n,sec}^2$$  \hspace{1cm} (47)

Large capacitance to volume ratio and low cost make aluminum electrolytic capacitors a suitable choice for the input/output capacitor. One has to parallel sufficient number of output capacitors to share ripple current. Low ESR series output capacitor is preferred to avoid excessive power dissipation. Such excessive power dissipation results in significant life degrading. The power dissipations of input/output capacitors are given by

$$P_{C_{in}} = \frac{I_{2rip,in}^2 R_{C_{in}}}{n_{C_{in}}}$$  \hspace{1cm} (48)

$$P_{C_{out}} = \frac{I_{2rip,out}^2 R_{C_{out}}}{n_{C_{out}}}.$$  \hspace{1cm} (49)

IV. OPTIMIZATION PROCEDURES

An optimization procedure is presented in this section. The optimization program in this paper is developed under MATLAB environment. The LLC efficiency optimization involves nonlinear, constrained, continuous optimization problems. The $\text{fmincon}(x)$ function of MATLAB optimization toolbox is applied as the optimizer to solve such problems. The “active-set” algorithm is used in the $\text{fmincon}(x)$ function. Detailed
optimization procedures can be found in [26]. The aim of the optimization is to minimize the loss at a certain loading condition. The flow chart of the optimization procedure is presented in Fig. 10. The characteristics of the power components are discrete, such as the primary MOSFETs, transformer core and bobbin size. The continuous optimization methods cannot handle such discrete values, so we pre-select the discrete components at the discrete component selection stage. In the continuous optimization stage, the discrete components and their related parameters are fixed.

Let \( x \) denote a vector containing all the design variables, such as switching frequency, primary turns, secondary turns, value of \( L_r, L_m, \) and \( C_r, \) etc.

\[
x = [f_s, n_p, n_s, L_m, C_r, L_r, d_{AWG}, n_{layer}, h_{foil}, n_{Lr}, d_{AWG}_{Lr}, n_{Lr_{layer}}]
\]  

(50)

The objective function \( P_{\text{loss}}(x) \) is the converter loss at full load condition. The optimization problem is to minimize the loss \( P_{\text{loss}}(x) \) subject to constraints set \( \Omega \), given by

\[
\min_{x \in \Omega} P_{\text{loss}}(x)
\]  

(51)

where \( \Omega \) is given by

\[
\Omega = \{ x | b_{x1} \leq x \leq b_{xu}, 0.3 - \Delta B_m, L_r \geq 0, 0.3 - \Delta B_m, X_F \geq 0, V_{\text{out}} = 12 \}.
\]  

(52)

The lower bound vector \( b_{x1} \) and the upper bound vector \( b_{xu} \) of design variables give the searching range, where the expression “\( x \geq b_{xj} \)” denotes “\( x - b_{xj} \)” to be a vector with non-negative entries. “\( 0.3 - \Delta B_m, L_r \geq 0 \)” and “\( 0.3 - \Delta B_m, X_F \geq 0 \)” denote that the resonant inductor and the transformer do not saturate (0.3 is assumed to be the ferrite flux saturation level).

The output voltage \( V_{\text{out}} \) given by (53) is required to satisfy the equality constraint and given by

\[
V_{\text{out}} = \frac{n_s M V_{\text{in}}}{2 n_p}.
\]  

(53)

This means that the output voltage should be regulated at 12 V. This equality constraint is nonlinearly related to many design variables, such as \( f_s, n_p, n_s, L_r, C_r, L_m, \) and the operation mode. The optimizer algorithm searches the optimum result that satisfies the constraint set.

V. OPTIMIZATION AND EXPERIMENTAL RESULTS

A. Optimization Results

The optimization program aims to optimize a 400 V input voltage, 12 V output voltage, and 25 A output current LLC resonant converter. The optimized design variables and lower/upper bounds are presented in Table III. The lower/upper bounds are predefined. It can be seen that some of the design variables converge to their boundaries. These boundaries are limited by physical factors such as size. This table also indicates those boundaries that can be improved to have even higher efficiency.

The loss distributions are also presented in Table IV. The optimized efficiency is calculated to be 97.4%, where the calculated efficiency is \( (\text{output power}) / (\text{all losses} + \text{output power}) \).

The output voltage \( V_{\text{out}} \) is required to satisfy the equality constraint and given by

\[
V_{\text{out}} = \frac{n_s M V_{\text{in}}}{2 n_p}.
\]  

(53)
Fig. 11. Calculated waveforms at optimized efficiency.

Fig. 12. Prototype converter and thermal images. (a) Prototype LLC converter. (b) Thermal image of prototype converter.

Fig. 13. Experimental waveform of prototype converter.

thermal design for components. The table can also direct the choice of individual components. The calculated waveforms of the optimized converter are presented in Fig. 11. The optimized LLC converter operates in CCMA and the switching frequency is only slightly higher than resonant frequency. The results are very similar to previous research works [7], [8] that good efficiency of LLC converter occurs at resonant frequency.

B. Experimental Results

A prototype LLC converter is built as shown in Fig. 12(a). The converter is designed to be naturally cooled. The circuit parameters are listed in Table V. The synchronous rectifier modules are placed vertically with the heat sinks in the original design. However, it is placed horizontally and the heat sinks are removed in order to take thermal inferred images. The thermal infrared image of the prototype converter operating at full load 12 V 25 A (for 2 h) is also shown in Fig. 12(b). The ambient temperature is 25 °C and there is no air flow. The prototype LLC converter operates at CCMA during full-load conditions, shown in Fig. 13. The Ch1 of Fig. 13 is the drain-source voltage of primary low side MOSFET. Ch2 and Ch3 are the drain-source voltages of secondary SRs. Ch4 is the resonant inductor current. The measured efficiency is 97.07% at full load where the efficiency is calculated by [(output voltage)*(output current)]/[(input voltage)*(input current)]. The average input voltage, input current, and output voltage are measured by the DC voltage meters. The average output current is measured by the DC current meter of the electronic load. The efficiency curve of prototype converter is shown in Fig. 14. The 50%-load-efficiency is higher than 97% and 20%-load-efficiency is higher than 96%. The calculated full-load loss is 7.99 W and the experimental full-load loss is (input power)-(output power), which is 9.05 W. The error of loss calculation is 11.8%, which is calculated by [(experimental loss)-(calculated loss)]/(experimental loss).

TABLE V

<table>
<thead>
<tr>
<th>Component</th>
<th>List</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synchronous rectifier</td>
<td>BSC016N04LS3</td>
</tr>
<tr>
<td>Isolation transformer</td>
<td>ET4D4/21/15 3C90</td>
</tr>
<tr>
<td>Resonant inductor</td>
<td>70µH</td>
</tr>
<tr>
<td>Resonant Capacitor</td>
<td>47 nF 1000 V 1.72K @ 2000 V</td>
</tr>
<tr>
<td>Input Capacitor</td>
<td>100µF 450 V</td>
</tr>
<tr>
<td>Output Capacitor</td>
<td>6*1800µF 16 V</td>
</tr>
<tr>
<td>Optimized LLC resonant converter</td>
<td>V_in = 400 V, V_out = 12 V, I_out = 25 A</td>
</tr>
<tr>
<td>Primary MOSFET</td>
<td>IPPS0R1440CP</td>
</tr>
<tr>
<td>Current driven SRs</td>
<td>AWG44*60</td>
</tr>
<tr>
<td>N_L=12</td>
<td></td>
</tr>
</tbody>
</table>

The table can also direct the choice of individual components. The calculated waveforms of the optimized converter are presented in Fig. 11. The optimized LLC converter operates in CCMA and the switching frequency is only slightly higher than resonant frequency. The results are very similar to previous research works [7], [8] that good efficiency of LLC converter occurs at resonant frequency.
VI. CONCLUSION

In this research work a systematic optimization procedure is proposed to optimize the LLC converter full-load efficiency. A mode solver technique is proposed to handle the LLC converter steady-state solutions. The mode solver utilizes numerical nonlinear programming techniques to solve LLC state equations and determine the operation mode. Loss models are provided to calculate the total component losses using the current and voltage information derived from the mode solver. The calculated efficiency serves as the objective function to optimize the converter efficiency. A prototype 300-W 400-V to 12-V LLC converter is built using the optimization results. The details of design variables, boundaries, equality/inequality constraints, and loss distributions are given. A measured full-load efficiency of 97.07% is achieved compared to the calculated 97.4% efficiency. The proposed optimization procedure is an effective way to design high-efficiency LLC converters.

REFERENCES

Godwin Kwn Yuan Ho received the B.Eng degree in electrical engineering from the University of Hong Kong, Hong Kong, in 2010, and is currently pursuing the M.Phil degree at the Power Electronics Laboratory, University of Hong Kong.

Bryan Man Hay Pong (M’84–SM’96) was born in Hong Kong. He received the B.Sc. degree in electronic and electrical engineering from the University of Birmingham, Birmingham, U.K., in 1983, and the Ph.D. degree in power electronics from Cambridge University, Cambridge, U.K., in 1987.

He worked with National Semiconductor Hong as a Senior Design Engineer and then a Chief Design Engineer. He also worked with ASTEC International as a Principal Engineer and a Division Engineering Manager. He is now an Associate Professor at the University of Hong Kong, Hong Kong. He is In Charge of the Power Electronics Laboratory. His research interests include high efficiency and high reliability power conversion, EMI reduction techniques, magnetic components, and other aspects of switch mode power conversion. He has co-invented a number of patents. He also works with CET Opto Co. Ltd. on power conversion and lighting products.

Bingo Wing-Kuen Ling (M’08–SM’08) received the B.Eng. (Hons) and M.Phil. degrees from the Department of Electronic and Computer Engineering, the Hong Kong University of Science and Technology, in 1997 and 2000, respectively, and the Ph.D. degree from the Department of Electronic and Information Engineering, the Hong Kong Polytechnic University, in 2003.

In 2004, he joined the Kings College, London, as a Lecturer. He has published more than fifty journal papers. He is also the author of the textbook titled *Nonlinear Digital Filters: Analysis and Applications* (Elsevier, 2007), and has edited a book titled *Control of Chaos in Nonlinear Circuits and Systems* (World Scientific Publishing, 2009). His research interests include theory and applications of optimizations, symbolic dynamics, filter banks, and wavelets as well as control theory. He has served as a Technical Committee member of several IEEE international conferences as well as an Organizer of a special session in the International Symposium on Communication Systems, Networks and Digital Signal Processing, in 2008 and 2010. He has also served as a Guest Editor of a special issue on nonlinear circuits and systems in the *Journal of Circuits, Systems and Signal Processing* and on optimization for signal processing and communications in the *American Journal of Engineering and Applied Science*. He is currently a Guest Associate Editor of the *International Journal of Bifurcations and Chaos*, and an Associate Editor of *Circuits, Systems and Signal Processing*.

James Lam (M’89–SM’99) received a first class B.Sc. degree in mechanical engineering from the University of Manchester, U.K., in 1983. He received the M.Phil. and Ph.D. degrees from the University of Cambridge, Cambridge, U.K., in 1985 and 1988, respectively.

He has been a Distinguished Visiting Fellow of the Royal Academy of Engineering. Prior to joining the University of Hong Kong in 1993, he was a Lecturer at the City University of Hong Kong and at the University of Melbourne. He has held Guest Professorships in many universities in China. He has research interests in model reduction, robust control and filtering, delay, singular systems, Markovian jump systems, multidimensional systems, networked control systems, vibration control, and biological networks.

Prof. Lam was awarded the Ashbury Scholarship, the A.H. Gibson Prize, and the H. Wright Baker Prize for his academic performance. He was a recipient of the Outstanding Researcher Award of the University of Hong Kong. He is a corecipient of the International Journal of Systems Science Prize Paper Award. He is a Chartered Mathematician, Chartered Scientist, a fellow of the Institute of Mathematics and its Applications, and a fellow of the Institution of Engineering and Technology. He is a Panel Member (Engineering) of the Research Grants Council, HKSAR. In addition to serving as Subject Editor of the *Journal of Sound and Vibration*, he is also Associate Editor of *Asian Journal of Control*, *International Journal of Systems Science*, *International Journal of Applied Mathematics and Computer Science*, *IEEE Transactions on Signal Processing*, *Journal of the Franklin Institute*, *Automatica, Multidimensional Systems and Signal Processing*, and is an editorial member of *IET Control Theory and Applications, Dynamics of Continuous, Discrete and Impulsive Systems: Series B (Applications & Algorithms)*, and Proc. IMechE Part I, *Journal of Systems and Control Engineering*. He formerly served as Editor-in-Chief of the *IEE Proceedings: Control Theory and Applications* and was a member of the IFAC Technical Committee on Control Design.