A New Image Fusion Algorithm Based on Wavelet Transform and the Second Generation Curvelet Transform

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Abstract—This paper analyzes the characteristics of the Second Generation Curvelet Transform and put forward an image fusion algorithm based on Wavelet Transform and the Second Generation Curvelet Transform. We looked at the selection principles about low and high frequency coefficients according to different frequency domain after Wavelet and the Second Generation Curvelet Transform. In choosing the low-frequency coefficients, the concept of local area variance was chosen to measuring criteria. In choosing the high-frequency coefficients, the window property and local characteristics of pixels were analyzed. Finally, the proposed algorithm in this article was applied to experiments of multi-focus image fusion and complementary image fusion. According to simulation results, the proposed algorithm hold useful information from source multiple images quite well.

Keywords—Image Processing, Image Fusion, Wavelet Transform, Second Generation Curvelet Transform, Regional Activity

I. INTRODUCTION

Image fusion is a data fusion technology which keeps images as main research contents. It refers to the techniques that integrate multi-images of the same scene from multiple image sensor data or integrate multi-images of the same scene at different times from one image sensor [1].

The image fusion algorithm based on Wavelet Transform which faster developed was a multiresolution analysis image fusion method in recent decade [2]. Wavelet Transform has good time-frequency characteristics. It was applied successfully in image processing field [3]. Nevertheless, its excellent characteristic in one-dimension can’t be extended to two-dimension or multi-dimension simply. Separable wavelet which was spanning by one-dimensional wavelet has limited directivity [4].

Aiming at these limitation, E. J. Candes and D. L. Donoho put forward Curvelet Transform theory in 2000 [5]. Curvelet Transform consisted of special filtering process and multi-scale Ridgelet Transform. It could fit image properties well. However, Curvelet Transform had complicated digital realization, includes sub-band division, smoothing block, normalization, Ridgelet analysis and so on. Curvelet’s pyramid decomposition brought immense data redundancy [6]. Then E. J. Candes put forward Fast Curvelet Transform(FCT) that was the Second Generation Curvelet Transform which was more simple and easily understanding in 2005[7]. Its fast algorithm was easily understood. Li Huihui’s researched multi-focus image fusion based on the Second Generation Curvelet Transform [8]. This paper introduces the Second Generation Curvelet Transform and uses it to fuse images, different kinds of fusion methods are compared at last. The experiments show that the method could extract useful information from source images to fused images so that clear images are obtained.

II. CURVELET TRANSFORM

Curvelet Transform was proposed by Cands and Donoho in 2000, it derived from Ridgelet Transform. They constructed a new Curvelet frame in 2005, it didn’t bring Ridgelet Transform different from traditional Curvelet Transform, but gave expression forms of Curvelet basis in the frequency domain; it was true Curvelet Transform.

A. Continuous Curvelet Transform

In two-dimensional space \( R^2 \), \( x \) stands for spatial domain variable, \( \varphi \) stands for frequency domain variable, \( r, \theta \) express polar coordinates. First a couple of window functions should be brought, \( W(r) \) and \( V(t) \) separately express radius window and corner window, \( W \) is supported in \( r \in (1/2, 2) \), \( V \) is supported in \( t \in [-1,1] \), then permitting condition should be satisfied:

\[
\sum_{r} W(2^{j}r) = 1, \quad r \in (3/4, 3/2); \\
\sum_{t} V^2(t - \ell) = 1, \quad t \in (-1/2, 1/2);
\]

To all scales \( j \geq j_0 \), frequency window of Fourier frequency domain is expressed as follows:

\[
U_j(r, \theta) = 2^{-j/4}W(2^{-j}r)V\left(\frac{2^{j/4}\theta}{2\pi}\right)
\]
\[ \left\lfloor j/2 \right\rfloor \] stands for \( j/2 \) rounding operation. There are differences of dilation factor between \( \mathbb{W} \) and \( \mathbb{V} \). In time domain, dilation factor of \( \mathbb{W} \) is \( 2^{-\ell} \) shorter, dilation factor of \( \mathbb{V} \) is \( 2^{\ell/2} \) longer, that is width = length², it is also called anisotropy scaling relation. \( \mathcal{U}/j \) stands for a wedge window in polar coordinates, it is expressed such as Fig.1:

![Fig.1 Example of an image with acceptable resolution](image)

Then Curvelet \( \varphi_j(x) \) is defined by \( \tilde{\varphi}_j'(w) = U_j(w) \), that is Mother Curvelet.

There are three parameters in Curvelet, \( j \) stands for scale, \( \ell \) stands for direction, \( k = (k_1, k_2) \in \mathbb{Z}^2 \) stands for space position. When the scale is \( 2^{-\ell} \), \( \theta = 2\pi \cdot 2^{-\ell/2}, \ell \), \( \ell = 0, 1, \ldots \), \( 0 \leq \theta \leq 2\pi \), translation position could be expressed as \( x_{j,\ell} = R_\theta^{-1}(k_1, 2^{-\ell}, k_2, 2^{-\ell/2}) \), then Curvelet could be expressed as follows:

\[
\varphi_{j,\ell,k}(x) = \varphi_j(R_\theta(x - x_{j,\ell}))
\]

(4)

In which, \( R_\theta \) is a rotation of \( \theta \), then all Curvelets in scale \( 2^{-\ell} \) could be obtained by rotation and translation of \( \varphi_j \).

The coefficients of Curvelet can be defined by inner product of \( f \in L^2(\mathbb{R}^2) \) and \( \varphi_{j,\ell,k} \):

\[
c(j,\ell,k) = \langle f, \varphi_{j,\ell,k} \rangle = \int_{\mathbb{R}^2} f(x) \overline{\varphi_{j,\ell,k}(x)} \, dx
\]

(5)

Reconstruction formula is

\[
f = \sum_{j,\ell,k} \langle f, \varphi_{j,\ell,k} \rangle \varphi_{j,\ell,k}
\]

(6)

Curvelet Transform should suit Parseval relation:

\[
\sum_{j,\ell,k} \| f, \varphi_{j,\ell,k} \|^2 = \| f \|^2_{L^2(\mathbb{R}^2)}
\]

(7)

B. Discrete Curvelet Transform

In two-dimensional Cartesian coordinate system, we use \( \mathcal{U}/j \) as a block region with the same center to replace it (see fig 2).

Then local window in Cartesian coordinate system is expressed as:

\[
\tilde{U}_j(w) = W_j(w) V_j(w)
\]

(8)

Here,

\[
\begin{align*}
\tilde{W}_j(w) &= \sqrt{\varphi_{j,\ell}^2(w) - \varphi_j^2(w)} \quad j \geq 0 \\
V_j(w) &= V(2^{-\ell/2}w_2/w_1)
\end{align*}
\]

(9)

\( \varphi \) is defined by inner product of one-dimensional lowpass window:

\[
\varphi_j(w_1, w_2) = \varphi(2^{-\ell}w_1, 2^{-\ell}w_2)
\]

(10)

We introduces slopes with the same interval,

\[
tan \theta_j = \ell \cdot 2^{-\ell/2} \quad \ell = -2^{-\ell/2}, \ldots, 2^{-\ell/2} - 1
\]

(11)

And the Discrete Curvelet Transform is defined as follows:

\[
c(j,\ell,k) = \int f(w) \tilde{U}_j(S_{\theta_j}^k(\Pi(x - S_{\theta_j}^k b))) \, dw
\]

(12)

Here, \( k \) quantizes \( (k_1 \times 2^{-\ell}, k_2 \times 2^{-\ell/2}) \), and the block \( S_{\theta_j}^k( k_1 \times 2^{-\ell}, k_2 \times 2^{-\ell/2}) \) is not a standard rectangle, so Fast Fourier Transform algorithm can’t be used, then rewrite the last formula to the following one:

\[
c(j,\ell,k) = \int f(w) \tilde{U}_j(S_{\theta_j}^k w) \exp(i \langle b, S_{\theta_j}^k w \rangle) \, dw
\]

(13)

Because the block \( S_{\theta_j}^k( k_1 \times 2^{-\ell}, k_2 \times 2^{-\ell/2}) \) is the wedgebase to approach the singularity of \( C^2 \), thus isotropic will be expressed; geometry of singularity is ignored. Curvelet Transform takes wedgebase to approach the singularity of \( C^2 \). It has angle directivity compared with Wavelet, and anisotropy will be expressed. When the direction of approachable baseline matches the geometry of singularity characteristics, Curvelet coefficients will be bigger[10].

First, we need pre-processing, then cut the same scale from awaiting fused images according to selected region. Subsequently, we divide images into sub-images which are different scales by Wavelet Transform. Afterwards, local Curvelet Transform of every sub-image should be

III. IMAGE FUSION ALGORITHM BASED ON WAVELET AND THE SECOND GENERATION CURVELET TRANSFORM

Images can be fused in three levels, namely pixel level fusion, feature level fusion and decision level fusion [9]. Pixel level fusion is adopted in this paper. We can take operation on pixel directly, and then fused image could be obtained. We can keep as more information as possible from source images. Because Wavelet Transform takes blockbase to approach the singularity of \( C^2 \), thus isotropic will be expressed; geometry of singularity is ignored. Curvelet Transform takes wedgebase to approach the singularity of \( C^2 \). It has angle directivity compared with Wavelet, and anisotropy will be expressed. When the direction of approachable baseline matches the geometry of singularity characteristics, Curvelet coefficients will be bigger[10].
taken, its sub-blocks are different from each others on account of scales’ change. The steps of using Curvelet Transform to fuse two images are as follows:

- Resample and registration of original images, we can correct original images and distortion so that both of them have similar probability distribution. Then Wavelet coefficient of similar component will stay in the same magnitude.
- Using Wavelet Transform to decompose original images into proper levels. One low-frequency approximate component and three high-frequency detail components will be acquired in each level.
- Curvelet Transform of individual acquired low-frequency approximate component and high-frequency detail components from both of images, neighborhood interpolation method is used and the details of gray can’t be changed.
- According to definite standard to fuse images, local area variance is chose to measure definition for low-frequency component. First, divide low-frequency \( C_{h}(k_{1},k_{2}) \) into individual foursquare sub-blocks which are \( N_{i} \times M_{i} \ (3 \times 3 \ or \ 5 \times 5) \), then calculate local area variance of the current sub-block:

\[
STD = \frac{1}{N_{i} \times M_{i}} \sum_{i=0}^{N_{i}-1/2} \sum_{j=0}^{M_{i}-1/2} (C_{h}(k_{1}+i,k_{2}+j)-\overline{C}_{h}(k_{1},k_{2}))^2
\]  

Here, \( \overline{C}_{h}(k_{1},k_{2}) \) stands for low-frequency coefficient mean of original images. If variance is bigger, it shows that the local contrast of original image is bigger, that means clearer definition. It is expressed as follows:

\[
\begin{align*}
C_{h}^{\ast}(k_{1},k_{2}) = \begin{cases} 
C_{h}^{\ast}(k_{1},k_{2}), & STD^4 \geq STD^a \ 
C_{h}^{a}(k_{1},k_{2}), & STD^4 < STD^a
\end{cases}
\end{align*}
\]  

(16)

Regional activity \( E_{ij}(k_{1},k_{2}) \) is defined as a fusion standard of high-frequency components. First, divide high-frequency sub-band into sub-blocks, then calculate regional activity of sub-blocks.

\[
E_{ij}(k_{1},k_{2}) = \frac{1}{N_{i} \times M_{i}} \sum_{i=-(N_{i}-1)/2}^{(N_{i}-1)/2} \sum_{j=-(M_{i}-1)/2}^{(M_{i}-1)/2} (C_{j}(k_{1}+i,k_{2}+j))^2
\]  

In which, \( N_{i} \times M_{i} \ means \ 3 \times 3, 5 \times 5 \ and \ so \ on.

- Inverse transformation of coefficients after fusion, the reconstructed images will be fusion images.

IV. EXPERIMENTAL RESULTS AND ANALYSIS

A. Multi-Focus Image Fusion

We use multi-focus lab images after standard testing. Fig.3(a) shows left-focus image, the outline of clock looks clear. Fig.3(b) shows right-focus image, the outline of person and the box printed 3M looks clear. Three fusion algorithms are adopted in this paper to contrast fusion effects. We separately use Discrete Wavelet Transform(DWT), the Second Generation Curvelet Transform that is Fast Curvelet Transform(FCT), Discrete Fast Curvelet Transform(DFCT) which is proposed in this paper. According to DWT and FCT, We use different fusion standard in different sections. Average operator is used as a fusion standard for low-frequency sub-band. Choosing the fusion operator based the biggest absolute value is used as a fusion standard for three high-frequency sub-band from the highest scale. Choosing the fusion operator based the biggest local area variance is used as a fusion standard for high-frequency sub-band from other scales. Fig.3(c), (d), (e) separately express corresponding fusion results.

Fig.3 shows that three algorithms all acquire good fusion results, focus difference has been eliminated, definition of original images have been proved. The result of DWT looks worse by contrast; we can see evident faintness in edges. False contours of edges appear in the FCT. We acquire the best subjective effect in DFCT. The fused image is the clearest, and detail information are kept as more.

We adopt Entropy of fused image, correlation coefficient \( C_{cc} \) and rms error \( E_{rms} \) [8] to evaluate the fused quality, it is expressed as table I. In the same group of experiments, if Entropy of fused image is bigger, or correlation coefficient approach one more closer, or \( E_{rms} \) is smaller. It shows that the fusion methods adopted is better.

**TABLE I. EVALUATION OF THE MULTI-FOCUS IMAGE FUSION RESULTS**

<table>
<thead>
<tr>
<th>Fusion Method</th>
<th>Evaluation of the multi-focus image fusion</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Entropy</td>
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<tr>
<td>DWT</td>
<td>4.3257</td>
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<tr>
<td>FCT</td>
<td>4.3962</td>
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<tr>
<td>DFCT</td>
<td>4.4742</td>
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</tbody>
</table>

Fig.3 Multi-focus lab images and their image fusion. (a):left-focus; (b):right-focus; (c):fused image of DWT; (d): fused image of FCT; (e): fused image of DFCT
B. Complementary Fusion Image

In medicine, CT and MRI image both are tomographic scanning images. The have different features. Fig.4(a) shows CT image, in which image brightness related to tissue density, brightness of bones is higher, and some soft tissue can’t been seen in CT images. Fig.4(b) shows MRI image, here image brightness related to amount of hydrogen atom in tissue, thus brightness of soft tissue is higher, and bones can’t been seen. There are complementary information in these images. We use three methods of fusion forenamed in medical images, and adopt the same fusion standards, Fig.4(c), (d), (e) separately shows results, the data of results is expressed as table II.

We make simulation experiments by above fusion methods in comparison. The results are expressed as Fig.4. Three algorithms all acquire good fusion results, in which the results of method in this paper have more detail information. The date from table II also shows the same conclusion.

<table>
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</tr>
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<tbody>
<tr>
<td></td>
<td>Entropy</td>
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<td>FCT</td>
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<tr>
<td>DFCT</td>
<td>3.7721</td>
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</table>

V. CONCLUSIONS

This paper puts forward an image fusion algorithm based on Wavelet Transform and the Second Generation Curvelet Transform. It includes multiresolution analysis ability in Wavelet Transform, also has better direction identification ability for the edge feature of awaiting describing images in the Second Generation Curvelet Transform. This method could better describe the edge direction of images, and analyzes feature of images better. According to it, this paper uses Wavelet and the Second Generation Curvelet Transform into fusion images, then makes deep research on fusion standards and puts forward corresponding fusion projects. At last, these fusion methods are used in simulation experiments of multi-focus and complementary fusion images. In vision, the fusion algorithm proposed in this paper acquires better fusion result. In objective evaluation criteria, its fusion characteristic is superior to traditional DWT and FCT’s.

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